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Investigating functional thinking in the elementary classroom: Foundations of early algebraic reasoning

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Abstract

This paper examines the development of student functional thinking during a teaching experiment that was conducted in two classrooms with a total of 45 children whose average age was nine years and six months. The teaching comprised four lessons taught by a researcher, with a second researcher and classroom teacher acting as participant observers. These lessons were designed to enable students to build mental representations in order to explore the use of function tables by focusing on the relationship between input and output numbers with the intention of extracting the algebraic nature of the arithmetic involved. All lessons were videotaped. The results indicate that elementary students are not only capable of developing functional thinking but also of communicating their thinking both verbally and symbolically.
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1. Introduction

Embedding algebraic reasoning in arithmetic reasoning has been the focus of recent research concerning elementary students. This represents a shift from the traditional approach of introducing algebraic reasoning after the development of arithmetic reasoning and is supported by research that suggests that algebraic reasoning occurs in conjunction with arithmetic reasoning (e.g., Brizuela & Schliemann, 2004; Kaput & Blanton, 2001; Warren & Cooper, 2001), perhaps even prior to the introduction of number (Dougherty, 2003). Classroom activities in the elementary years conventionally focus on mathematical products rather than on mathematical processes (Malara & Navarra, 2003). Arithmetical experiences tend to centre on the development and the use of particular relationships for computation. Traditionally, elementary schools have given little emphasis to relations and transformations as objects of study. This paper reports the results of a teaching experiment that commences to address a number of these concerns.

2. Learning and teaching functional thinking

Fundamental to relation and transformation is the concept of function, a schema about how certain quantities relate, or are changed or transformed, to other quantities (Chazan, 1996). A function is denoted or expressed in terms of the

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relationship between a first variable quantity and a second variable quantity or in terms of the change from the second to the first. One example of this is the total cost of bananas bought at the supermarket and the weight purchased. Functional thinking could be seen as representational thinking that focuses on the relationship between two or more varying quantities (Smith, 2003). Functions can be represented in a variety of ways other than algebraic expressions, namely, graphs, tables, arrow diagrams, as well as verbal descriptions. The verbal descriptions of these representations at the elementary level are called 'algebraic babbling' (Malara & Navarra, 2003) and are believed to be precursors for the precise use of mathematical language in the secondary school (Warren, 2005).

The construction and use of functions is considered to be central to most mathematical investigations and has been found to be notoriously difficult for most students at all levels of learning (Cuoco, 1995). From an epistemological perspective Dubinsky and Harel (1992) suggest that as one abstracts the notion of a function there appears to be three main landmarks that occur, namely, functions as actions, functions as processes, and functions as objects. Students who view a function as an action see a function as a set of isolated calculations and tend to refer to the function in terms of a particular input. As they chunk the individual steps together into a coherent whole and begin to interiorize these sequences the function becomes a process. Finally, as students suppress the details of the calculations of outputs and the focus changes from the calculations of outputs to the behaviour of the function itself, they are beginning to view the function as an object. It is at this stage that they can begin to compare functions with each other.

The study of mathematical change is fundamental to understanding functions and higher levels of mathematics (e.g., calculus) that are based on it (National Councils of Teachers of Mathematics, 2000). It not only serves higher levels of mathematics but also assists in a better understanding of the processes of arithmetic (Warren & Cooper, 2001). In particular, it assists in developing an understanding of the relationships between the operations, particularly the inverse relationship between addition and subtraction and multiplication and division, for example, *If my number is increased by 2 and is now 8, what was my original number?* (Warren & Cooper, 2001, 2005).

Research has indicated that many young adolescents experience difficulties in finding relationships between two data sets (Stacey & MacGregor, 1995; Warren, 1996; Warren, 2000). Approaches used to find the general rule, that is, defining one data set in terms of the other data set appear to fall into three broad categories (Redden, 1996). These are: (a) using one example to predict the relationship between uncountable examples (e.g., if 5 gives 10 then 20 gives 40), (b) the additive strategy where connections among consecutive elements (e.g., you add 3 onto each input number), and (c) functional strategy where a relationship is formed between the two data sets (e.g., the output number is 3 times the input number). It is conjectured that encouraging students to find the relationship for uncountable examples assists them to identify the correct functional relationship between the two sets of data (Redden, 1996; Warren, 1996).

The study of students' functional thinking indicates that its development should commence in the elementary years and, therefore, be gradual and occur over a long period of time. Function is not a concept that students typically understand (Chazan, 1996) and students have difficulty expressing themselves using algebraic notation (Neria & Amit, 2004). This lack of understanding and difficulty in expression is considered to be due to the abrupt and abstract way functions are commonly introduced in the high school. Research on children as young as 9 years shows that they can identify functional relations and use algebraic notation (Kaput & Blanton, 2001; Brizuela & Schliemann, 2004).

2.1. Theoretical framework

The theoretical stance that underpins the selection of materials, tasks and interactions between teacher and students, emerges from the literature relating to the construction of 'mental models'. Mental representations are believed to play a key role when students endeavour to think about mathematical situations (Davis & Maher, 1997). They become powerful tools with which students think.

The choice of representations or mental models is imperative. They need to (a) reflect the important relations and principles of the domain, (b) be unambiguous because students need to understand the structure of the representations and recognize the correspondence between the representations, (c) encompass different modalities including auditory, kinesthetic and visual, and (d) be situated in social practice because deep and transformative learning occurs when learners construct their own knowledge when it is meaningful (English, 1997; Presmeg, 1997; Rogoff, 1990). It is also well recognized that for the construction of student understanding, teaching should preferably be placed in the context of real world situations (Cobb, Jaworski, & Presmeg, 1996; Henningsen & Stein, 1997).

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