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Journal of Mathematical Behavior 26 (2007) 48–59

Mathematical Behavior

www.elsevier.com/locate/jmathb

Making sense of the traditional long division algorithm

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Abstract

This classroom scholarship report presents a group of elementary students' experiences learning the traditional long division algorithm. The traditional long division algorithm is often taught mechanically, resulting in the student's performance of step-by-step procedures with no or weak understanding of the concept. While noting some initial difficulties, the class episodes in this article provide examples of internalization that highlight the active role of the learner in transforming concrete representations into an abstract algorithm. Several factors encouraged students to be deeply engaged in making sense of the long division algorithm: meaningful tasks based on a theoretically well-articulated curriculum, effective pedagogical measures, and dynamic class discussions.

Keywords: Algorithms; Long division; Conceptual understanding; Elementary mathematics; Teaching strategies

Students' computation errors have been examined for several decades in many experimental studies (Ashlock, 2001; Brown & Burton, 1978; Brown & VanLehn, 1980; Burton, 1982; VanLehn, 1990). These studies endorsed the distinction between systematic errors and slips. Systematic errors, or bugs, result from the consistent application of a faulty method, algorithm, or rule, whereas slips are unsystematic careless errors. One of the widely shared explanations for students' systematic computation errors is their flawed or weak understanding of the place value system. When the procedural aspect of computation is overemphasized without clear conceptual understanding of the place value system, students tend not to think about the meaning of what they are doing and simply parrot someone else's directions in order to perform calculations (O'Brien, 1999).

The traditional long division algorithm is one familiar example that many students find particularly difficult to perform with understanding. Some features of the traditional long division algorithm create more difficulties for students than any of the other algorithms for basic operations. For example, unlike other operations, the traditional long division algorithm starts with the left-hand or larger place values; the required estimation skills often generate anxiety; and it is hard for some students to identify the magnitude of the answers that they are writing in each place (Fuson, 2003; Van De Walle, 2001). This situation often leads learners or teachers to nonsensical mnemonic phrases, such as "Does McDonald's Serve Breakfast?" or "Dirty Monkey Smells Bad" to memorize the sequence of "Divide–Multiply–Subtract–Bring it down." By memorizing these mnemonics, students might produce correct answers. However, many educators have asserted that the essence of doing mathematics is the process of "making sense" or "figuring out" (e.g., Schoenfeld, 1991; Skemp, 1987), and 'McDonald's Breakfast' or 'Dirty Monkeys' alone make no contribution to this process. Unfortunately, it seems that such an approach forges its way into the classroom with little resistance.

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doi:10.1016/j.jmathb.2007.03.001

Recently, many have shown that alternative or student-invented algorithms are more closely related to students' natural ways of solving problems (Randolph & Sherman, 2001) and eventually are more effective in enhancing students' understanding of numbers and operations (Carroll & Porter, 1998; Heuser, 2005). The current research will demonstrate that the traditional algorithms can be meaningfully taught if students have opportunities to engage in conceptually sound activities and to appreciate the meaning of algorithms at the early stage, instead of relying on the mechanical memorization. In the end, we must realize that algorithms are fully conceptual cultural historical products and should be taught as such (Schmittau, 2004).

This article exemplifies this point by providing actual classroom episodes of a group of primary grade students. The students actively engaged in the process of internalizing the conceptual meaning behind the traditional long division algorithm. The process was gradual and took about eight 1-hour class sessions. The students had extensive discussions on place value, the effective representations of their actions, and the transition from physical models to the written traditional long division algorithm. Thus, the purpose of this article is to present an example of how appropriate pedagogical tools and carefully selected activities can assist students in developing an understanding of the traditional long division algorithm and in attaining computational proficiency.

1. Background

1.1. Setting

The class episodes were taken from a 3-year classroom action research conducted in a private school located in an eastern state of the United States (Lee, 2002). This school implemented a Russian developmental mathematics curriculum developed by Davydov and his colleagues. The author taught the first 3 years of elementary curriculum to a group of students. This school is not a typical American school in terms of the number of students, class size, and curriculum. The philosophy of this school is rooted in a learning theory based on concepts developed by John Dewey and is evidenced in several ways: multi-aged and flexible grouping; child-centered, project-oriented tasks; a challenging curriculum, and written narratives as assessments.

1.2. Students

Seven students completed 3 years of Davydov's curriculum with the author. All of them were European–Americans. There were two male students and five female students. Two students had public school experience (2 years and 2 months, respectively), and for the others, this school was their first formal educational setting. The students' ages were between 8 and 10 years old at the time the long division algorithm was introduced, in our third year together.

1.3. Class environment

Davydov's curriculum is laid out as a series of problems. In a 1-hour class session, usually two to five problems were discussed. The majority of class time was spent in discussion. The selection of problems was based on the previous day's work or homework, or the introduction of new concepts.

In the class discussions, the students were asked to present their solution strategies to the given problems. If any of the classmates did not agree with the reasoning to a solution presented, the presenter needed to provide further justification and explanation. Class discussion continued until all the students agreed on the outcome.

1.4. Research method and documentation of class episodes

This classroom action research placed the emphasis on "the interpretations teachers and students are making and acting on in the situation" (Kemmis & McTaggart, 2000, p. 569). As Schon (1983) asserted, this research acknowledges practice as a kind of research where "inquiry is a transaction with the situation in which knowing and doing are inseparable" (p. 165). The class episodes reported in this article depict the class discussion process concerning the long division algorithm from a teacher/practitioner's perspective, focusing on the students' thinking and the teacher's pedagogical ideas and strategies.