Journal of Mathematical Behavior 26 (2007) 60–82

Mathematical Behavior

www.elsevier.com/locate/jmathb

Argumentation and participation in the primary mathematics classroom
Two episodes and related theoretical abductions

Götz Krummheuer *

Institut für Didaktik der Mathematik, Fachbereich Informatik und Mathematik, Johann Wolfgang Goethe Universität, Senkenberganlage 11, 60054 Frankfurt am Main, Deutschland, Germany

Available online 17 April 2007

Abstract

The main assumption of this article is that learning mathematics depends on the student's participation in processes of collective argumentation. On the empirical level, such processes will be analyzed with Toulmin's theory of argumentation and Goffman's idea of decomposition of the speaker's role. On the theoretical level, different statuses of participation in processes of argumentation will be considered. By means of the method of comparative analysis, different grades of autonomy according to the interactional contribution of a student can be reconstructed. The paper finishes with remarks about consequences for improving mathematics teaching in schools and mathematics teacher education at university level.

Keywords: Abduction; Argumentation; Comparative analysis; Interactional theory of learning mathematics; Participation; Production-design; Social interaction

0. Introduction

Obviously, mathematics education as a scientific discipline has to do with the ongoing demand of changing mathematics education in real school situations. Very frequently in the United States, these innovative approaches are described as the dichotomy of "traditional" versus "reform". Researchers work out ways in which their approach meets reform suggestions and reform standards as set out by the National Council of Teachers of Mathematics. In Germany we also have broad discussions about new standards for mathematics teaching and learning, evolving into a wider public concern due to the inadequate performance of German pupils on mathematics achievement tests in recent large scale studies TIMSS and PISA. A major change in teaching mathematics has been demanded.

The strategy for improving mathematics teaching is usually divided into three phases: first, the formulation of an innovative idea, second the development of a product – the new curriculum – and, third, the manner of its implementation. This three-phase model has already been applied in the 1960s and 1970s under acronyms like R-D-D or R-D-D&E, which stand for research-development-dissemination and evaluation. As Howson, Keitel, and Kilpatrick (1981) mentioned, this model was adapted from industrial innovation:

* Tel.: +49 69 798 28399; fax: +49 69 798 25195.
E-mail address: krummheuer@math.uni-frankfurt.de.

0732-3123/$ – see front matter © 2007 Elsevier Inc. All rights reserved.
doi:10.1016/j.jmathb.2007.02.001

> The early curriculum development projects borrowed their strategies, consciously or not, from the procedures used by industries in developing new products (p. 79).

Needless to say, many of these approaches failed, and a concern is that in our recent attempts to improve mathematics teaching we tend to repeat the mistakes and failures of the reform-period of the 1960s and 1970s. One major problem of those times of curriculum reform was the strict focus on the outcome, i.e., on the success. As Miles (1964) already criticized in the 1960s:

> The dominant focus in most contemporary change efforts, however, tends to be on the content of the desired change, rather than on the features and consequences of change processes. It is the thesis . . . that attention to change processes is crucial (p. 2).

Fortunately, we can look back on research of the last decades for the purpose of analyzing classroom processes. It is mainly from these studies – based on videotapes taken in regular classroom sessions – that we found new insights into the regularities of classroom interaction. Bauersfeld's article about the "Hidden dimensions of the so-called classroom reality" (Bauersfeld, 1980) provides central insights into aspects of classroom life, which expand Miles claim about the "attention to change processes" in two ways. First, it is of interest to study not only implemented change processes, but also regular teaching situations which, while making no claim to implement an innovative teaching approach, can also include potential for change. Thus, the main focus is on the analysis of *process* and not *product*. Second, when analyzing such processes, one discovers a certain domain of reality, which is somehow just between the sociological level of institutionalized aspects of school and its reform, and the psychological level of the cognizing individual. There are two possible reasons that Bauersfeld describes this intermediate domain as "hidden dimensions":

- theoretically, the mainstream theories in mathematics education, like the psychology of learning or curriculum theory, do not have concepts, niches or both for these dimensions;
- practically, most of these processes, which he called "hidden", are not consciously presented in the cognitions of the participants.

Schütz and Luckmann (1979) characterize this domain of reality as the "everyday world" (Lebenswelt; p. 25). With regard to my field of research, I speak of "everyday mathematics classroom situations", defined as the mutually referring actions of the participants of a social event in a mathematics class. These mutually referring actions create an *interactional arena*. This is the immediate zone of adaptation, action, planning and experience (Soeffner, 1989, p. 12) for the students and the teacher. Its feature is to be understood as "situational" as Goffman (1974) states:

> My perspective is *situational*, meaning here a concern for what one individual can be alive to at a particular moment, this often involving a few other particular individuals and not necessarily restricted to the mutually monitored arena of a face-to-face gathering (p. 8, italics added).

The term "situational" not only refers to what is specific to a situation, which is referred to as "situated", but above all, "situational" refers to what only can happen in the social interaction among people. So, for example, if, during a lesson, a student is working on his worksheet by himself, this may be a "situated" learning process (Lave & Wenger, 1991), defined by the "practice" he gained in the past in dealing with the kind of tasks written on the sheet. His action changes into a situational process, if the student takes up contact with others present in the classroom. This interaction flows into his dealing with the task, however rudimentary the process might be. Thus, the arena of interaction is accomplished by the interactional moves of the participants. Whether a teaching approach that one could classify as "meeting the reform standards" emerges in such a process has very much to do with situational decisions and interactional moves of the participants, including the teacher as well as the students. These decisions and moves are part of the dynamics of the everyday classroom situation.

With respect to the spontaneous and open development of the process of interaction, in this arena, traditional teaching and reform-teaching lie very close together. Often it is only a small move that shifts a process into traditional or reform paths. Thus, innovation of an everyday mathematics classroom situation emerges from the inside of this interactional arena. As we learned from Bauersfeld, many aspects of these everyday mathematics classroom situations are hidden in both senses described above.