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Longitudinal mathematics development of students with learning disabilities and students without disabilities: A comparison of linear, quadratic, and piecewise linear mixed effects models ☆

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ABSTRACT

Effective instructional planning and intervening rely heavily on accurate understanding of students' growth, but relatively few researchers have examined mathematics achievement trajectories, particularly for students with special needs. We applied linear, quadratic, and piecewise linear mixed-effects models to identify the best-fitting model for mathematics development over elementary and middle school and to ascertain differences in growth trajectories of children with learning disabilities relative to their typically developing peers. The analytic sample of 2150 students was drawn from the Early Childhood Longitudinal Study — Kindergarten Cohort, a nationally representative sample of United States children who entered kindergarten in 1998. We first modeled students' mathematics growth via multiple mixed-effects models to determine the best fitting model of 9-year growth and then compared the trajectories of students with and without learning disabilities. Results indicate that the piecewise linear mixed-effects model captured best the functional form of students' mathematics trajectories. In addition, there were substantial achievement gaps between students with learning disabilities and students with no disabilities, and their trajectories differed such that students without disabilities progressed at a higher rate than their peers who had learning disabilities. The results underscore the need for further research to understand how to appropriately model students' mathematics trajectories and the need for attention to mathematics achievement gaps in policy.

1. Introduction

Mathematical competence is a basic educational objective for all learners (Bryant & Bryant, 2008) given its implications for educational achievement, economic outcomes, and adaptive functioning throughout adulthood (Siegler et al., 2012). In particular, early competencies have long-term consequences for students' mathematics development and functional skills (Geary, 2013). In an era of increasing emphasis on science, technology, engineering, and mathematics (STEM) education, many scholars identify mathematics achievement and achievement gaps as issues of national significance (Brown-Jeffy, 2009). Despite continued emphasis

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on STEM development in policy, research, and practice at all levels of education, race-based mathematics achievement gaps appear to be increasing (Stinson, 2006), underscoring the need for greater attention to these gaps and mathematics instruction and intervention. Accurate knowledge of students' mathematics development may help educators improve the content, pacing, and delivery of mathematics instruction and interventions. Longitudinal studies are especially important to understanding the nature and extent of achievement gaps and differences in growth rates. Our understanding of mathematics achievement trajectories, however, is less developed than research on reading (Watson & Gable, 2012). Few studies have examined mathematics trajectories over several years, and fewer still have included students with disabilities (Jordan, Kaplan, & Hanich, 2002). The purpose of this study was to examine trajectories in mathematics achievement over the course of elementary and middle school to ascertain (a) the best-fitting functional form (i.e., shape) underlying the developmental process of mathematics achievement over time and (b) the nature and extent of gaps between students with learning disabilities (LDs) and their typically developing peers over time.

1.1. Mathematics development

Humans are predisposed to developing basic arithmetic skills, but most skills, particularly higher-order ones, are fostered through schooling (Shalev, 2004). Mathematical proficiency is not an isolated skill set; instead, it involves memory, language, reasoning, spatial skills, processing speed, and phonological processing (Butterworth, Varma, & Laurillard, 2011; Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012). Comprehension of magnitude and numerosity emerges during infancy and mathematical development continues throughout childhood and adolescence (Landerl & Kölle, 2009). Most typically developing children progress from counting and calculations up to three as toddlers, to recognizing arithmetic symbols and doing basic addition and subtraction by age 8, and to acquiring of multiplication and division, along with triple-digit arithmetic, between ages 9 and 12 (Shalev, 2004). For the average child, the foundations of numeracy and arithmetic are typically established by third grade (Compton et al., 2012), but mathematics proficiency is not fully attained until after elementary school (Baumert, Nagy, & Lehmann, 2012).

Mathematics knowledge and skills become increasingly complex as most students move through school and build on the foundational learning from the primary grades (Shin, Davison, Long, Chan, & Heistad, 2013). On average, growth appears to slow in later grades (Shin et al., 2013), so differences in ability or educational opportunity may contribute to, and even exacerbate, achievement gaps because initial deficits compound and hinder responsiveness to later instruction such that initial differences lead to intractable gaps (Shin et al., 2013). For example, early math difficulties affect students' counting, acquisition of basic facts and number sense (Bryant & Bryant, 2008), and scholars posit that failure to master basic mathematics facts is a core component of chronic mathematics difficulties (Gersten, Jordan, & Flojo, 2005).

1.2. Mathematics difficulties and learning disabilities

Nearly a third of United States students demonstrate significant deficits in mathematics knowledge and skills (National Assessment of Educational Progress, 2009), and up to 8% have severe mathematics difficulties for which they receive legally-mandated special education services as students with LD (Geary, Hamson, & Hoard, 2000; Shalev, 2004). Students with LD comprise the majority of the 10% of United States students who receive special education for educational disabilities (Data Accountability Center, 2012). Both reading and mathematics deficits are common, and recent research suggests that the majority of students with LD have deficits in mathematics (Compton et al., 2012). Given the substantial proportion of students affected by LD and the effects of the disorder on both short and long-term outcomes, it is important for researchers to consider the unique learning needs and outcomes of this group.

Although mathematics difficulties are common among students with LD, considerably less is known about the manifestation of mathematics difficulties relative to reading problems or, consequently, appropriate instruction or intervention (Augustyniak, Murphy, & Phillips, 2005). Affected individuals generally demonstrate persistently low achievement in mathematics with deficits across mathematics cognition tasks (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). It remains unclear whether these difficulties are due to delays or deficits in neurocognitive processes, but mathematics learning disability (MLD) has been conceptualized as unexpected underachievement in mathematics or specific deficits in the underlying cognitive processes and selective functional neurological impairment (Compton et al., 2012). For instance, although scholars have suggested that children with LD in mathematics demonstrate qualitatively different processing in mathematics than their typical peers, other research indicates that these children do not process mathematics differently than their peers without mathematics difficulties but instead that their processing is immature and inefficient (Landerl & Kölle, 2009).

1.3. Mathematics achievement of students with LD

Although discussion of achievement gaps usually centers on racial disparities, persistent gaps are also present between students with and without disabilities; these gaps have been studied infrequently despite the implications of such inquiry for understanding learning differences and informing appropriately differentiated instruction and intervention. Researchers have called for more attention to the ways common trajectories of students with LD differ from their typically developing peers (Butterworth et al., 2011). In particular, scholars consider longitudinal modeling essential to understanding growth trajectories and gaps, and, in turn, LD, because it enhances our understanding of students' learning difficulties over time (Gersten et al., 2005; Mazzocco & Räsänen, 2013). Furthermore, comparison of different modeling approaches allows for determination of which models best fit students' trajectories.