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Adaptive number knowledge: Exploring the foundations of adaptivity with whole-number arithmetic

Jake McMullen [a,*], Boglárka Brezovszky [a], Gabriela Rodríguez-Aflecht [a], Nonmanut Pongsakdi [a], Minna M. Hannula-Sormunen [b], Erno Lehtinen [a]

[a] Department of Teacher Education and Centre for Learning Research, University of Turku, 20014, Finland
[b] Turku Institute for Advanced Studies and Department of Teacher Education, University of Turku, 20014, Finland

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ABSTRACT

Adaptivity with arithmetic problem solving is a key aspect in the long-term development of mathematical skills, and the knowledge of numerical characteristics and relations is a core component of adaptivity with whole-number arithmetic. The present studies represent the first attempt to investigate the nature of the adaptive number knowledge, referring to the well-connected knowledge of numerical characteristics and arithmetic relations between numbers, which underlies adaptivity with arithmetic. A new measurement tool, the Adaptive Number Knowledge Task, was developed to capture individual differences in primary school students' and adults' number knowledge. Three groups of participants – 3rd to 5th graders, university students, and 6th graders – completed the measure, which required participants to produce arithmetic sentences equaling a target number by using any combination of four to five given numbers and arithmetic operations. These studies revealed substantial individual differences in the quantity and quality of participants' adaptive number knowledge. Adaptive number knowledge was found to be related to arithmetic fluency and knowledge of arithmetic concepts in 6th graders. Results suggest that the Adaptive Number Knowledge Task was able to capture individual differences in recognizing and using numerical relations in whole-number arithmetic problem solving, and that these differences may help explain differences in adaptivity with arithmetic.

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1. Introduction

It is now well-established in research on mathematical development and mathematics education that teaching and learning must aim to develop adaptive expertise with mathematical competences (Baroody, 2003; Hatano & Oura, 2003; Heinze, Star, & Verschaffel, 2009). Within whole-number arithmetic, one key component of adaptive expertise is the mastery of selecting the most appropriate strategy for solving a given problem, referred to as adaptivity with arithmetic problem solving. Developing adaptivity with arithmetic has been identified as a cornerstone of later mathematical success, including success with algebra (Kieran, 1992). Thus, the teaching and development of adaptivity with arithmetic requires a great deal of attention.

Traditional teaching and assessment methods of students' arithmetic problem solving strategies often focus on a limited number of procedures (e.g. Siegler & Lemaire, 1997), restricting the scope of the skills and competences that are taught and measured (Baroody, 2003; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009). It has been suggested that this approach fails to take into account how the knowledge of mathematical connections between numbers and operations is necessary for recognizing opportunities to use strategies in problem solving (Threlfall, 2002, 2009). Having a rich network of knowledge about characteristics of numbers and the relations between numbers, which can be flexibly applied in solving novel arithmetic tasks, is a fundamental component to adaptivity with arithmetic. This type of numerical knowledge is herein referred to as adaptive number knowledge. In other words, when displaying a high level of adaptivity with arithmetic, the recognition and use of the relations between the numbers and operations in a given problem is done in the most advantageous way for that person, on that problem, in that situation. Therefore, when investigating individual differences in adaptivity with arithmetic, it is necessary to also investigate students' underlying awareness of the numerical features of the arithmetic problems that may be required for the adaptive use of problem solving strategies. With this in mind,

* Corresponding author at: Centre for Learning Research, University of Turku, 20014, Finland.
E-mail address: jake.mcmullen@utu.fi (J. McMullen).

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the present set of studies investigates, for the first time, individual differences in primary school students' and adults' adaptive number knowledge by exploring their use of numerical characteristics and relations in producing arithmetic sentences.

1.1. Adaptivity with arithmetic problem solving

For at least the past ten years, research on learning and instruction of arithmetical skills and competences has bent towards providing opportunities for the development of adaptive expertise with arithmetic, as opposed to routine expertise (Blöte, Klein, & Beishuizen, 2000; Hatano & Oura, 2003). Within the study of adaptive expertise with arithmetic, the focus has been on adaptivity and flexibility with arithmetic. Verschaffel et al. (2009) distinguished between the ability to switch between multiple strategies – referred to as flexibility – and the ability to choose the most appropriate strategy for solving a particular problem — adaptivity. Adaptivity involves choosing the most appropriate strategy for a given problem, based on problem characteristics, personal characteristics (e.g. own skill with particular methods), and the socio-cultural setting. Thus, expert mathematicians display more adaptivity by using a more diverse set of strategies in solving difficult mental arithmetic estimation tasks than novices. They also rely more heavily on specific features and relations of numbers to find "nice" numbers (e.g. recognizing that 59 is close to a multiple of 20), to use in their estimations and almost never using more inefficient standard algorithms (Dowker, 1992). More fluent use of multiple strategies has been found to be related to general mathematical ability (Rittle-Johnson & Star, 2007; Star & Seifert, 2006), and while those with mathematical learning disabilities are able to use multiple strategies (Peters, De Smedt, Torbeyns, Verschaffel, & Ghesquière, 2014), they use them less than typically developed peers (Kieran, 1992; Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). In all, adaptivity with arithmetic problem solving, as an indication of adaptive expertise with arithmetic, appears to be an important aspect of mathematical development.

More recent efforts have suggested the need to consider adaptive number knowledge when investigating the development of arithmetic skills. The adaptivity approach described by Verschaffel et al. (2009) already takes into account the role of knowledge of numerical relations as well as socio-constructivist developmental considerations. Verschaffel et al. (2009) also bring up the issue of working memory limitations when considering adaptivity in arithmetic problem solving. Relatedly, there are almost a limitless number of strategies that can be used to solve a task and these solutions must be recalled when choosing a strategy (Threlfall, 2002, 2009). This consideration, along with the working memory limitations calls into doubt whether it is even possible to choose from a finite list of problem solving strategies at all (Baroody, 2003; Threlfall, 2009). Instead, one possibility is that in every problem solving situation, strategies are developed and re-developed in-situ by approaching a solution from the bottom up using the numerical characteristics and relations available in the problem at hand (Threlfall, 2002, 2009), for example through the use of "nice" numbers (Dowker, 1992). One further indication of the need to address adaptive number knowledge is the disconnect between the existence of knowledge of problem solving strategies and the lack of use of these strategies (Blöte et al., 2000), especially when not guided to do so (Gaschler, Vaterrodt, Frensch, Eichler, & Haider, 2013; Haider, Eichler, Hansen, Vaterrodt, & Frensch, 2014). These results suggest that there might be a gap in students' ability to independently recognize when to use certain problem solving strategies, despite being able to use these strategies when externally guided to do so.

Even if this bottom up approach to strategy choice fails to adequately explain all facets of adaptivity with arithmetic strategies; it is apparent that knowledge of numerical characteristics and relations (e.g. multiples and factors, features of the base-ten system, etc.) is a key component of adaptivity that must be taken into account. For example, the efficient use of the indirect addition strategy (e.g. solving 61–59 by solving 59 + __ = 61?) to solve subtraction problems requires recognizing that the two numbers being subtracted are relatively close to one another, which requires awareness of the numerical characteristics of the base-ten system.

The role of adaptive number knowledge is especially apparent in how students use certain strategies on problems that have numerical relations that are more difficult to recognize. There are larger individual differences with the use of additive composition than commutativity (Canobi, Reeve, & Pattison, 2003; Dubé, 2014) suggesting that it may be more difficult to recognize opportunities to use additive composition, since it requires the recognition of numerical characteristics and relationships (e.g. in 43–5 + 7, seeing the opportunity to make a round number). Another example of the influence of adaptive number knowledge on strategy use is when second-grade students (Blöte et al., 2000) only used the 'short-jump' procedure ("bridging the difference" using intermediate numbers that are easily calculated) on those subtraction items where it was most appropriate, as when the given numbers straddled a round number (e.g. '82–79 = 3': '79 to 80' = 1, '80 to 82' = 2, '1 + 2 = 3'). More proficient problem solvers — both young and old — are also more likely to recognize opportunities to use numerical characteristics and relations to solve problems more efficiently (Dowker, 1992; Nys & Content, 2010; Torbeyns et al., 2009).

Based on these findings, adaptive number knowledge is theorized to play a key foundational role in adaptivity with arithmetic problem solving, alongside flexible strategy choice (Verschaffel et al., 2009). Fig. 1 describes the situation of adaptive number knowledge in relation to adaptivity, within the framework of adaptive expertise with arithmetic during primary school and beyond, after the introduction of formal arithmetic. As can be seen, both 'numerical knowledge and skills' and 'arithmetic knowledge and skills' are suggested to be necessary, but not sufficient, components of adaptive number knowledge. In order to have well-developed adaptive number knowledge, students must be aware of the nature of the natural number system (numerical knowledge) and be able to use this knowledge in arithmetical problem solving. This involves being able to determine arithmetical relations between numbers and recognize the key characteristics of numbers that are useful for flexible problem solving (such as having many factors, being close to other "nice" numbers, estimates of possible multiples, etc.). Concurrently, adaptive number knowledge requires well-developed arithmetical knowledge and skills. This involves knowledge

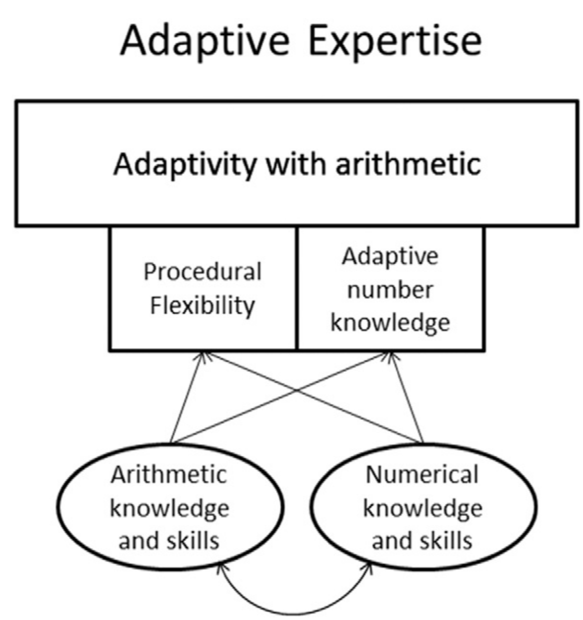

Fig. 1. Model of the components and foundations of adaptivity with arithmetic, within the framework of adaptive expertise with arithmetic problem solving.

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