Learning and Individual Differences 38 (2015) 10–17

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

The development of (non-)symbolic comparison skills throughout kindergarten and their relations with basic mathematical skills

Sylke W.M. Toll [a], Sietske Van Viersen [a,b], Evelyn H. Kroesbergen [a], Johannes E.H. Van Luit [a,*]

[a] *Department of Special Education, Utrecht University, P.O. Box 80.140, 3508 TC Utrecht, The Netherlands*
[b] *Department of Education and Learning Problems, University of Amsterdam, Nieuwe Prinsengracht 130, 1018 VZ Amsterdam, The Netherlands*

ARTICLE INFO

Article history:
Received 8 May 2014
Received in revised form 18 December 2014
Accepted 30 December 2014

Keywords:
Numerical development
(Non)symbolic comparison
Mapping
Math achievement
Kindergarten

ABSTRACT

Although numerical skills have proven to be important precursors for mathematical proficiency, longitudinal studies on numerical development are rather scarce. The overall goal of the present study is to gain insight in numerical skills, that is non-symbolic and symbolic comparison skills, as precursors of mapping skills and basic math achievement of children within a longitudinal design. Over two and a half years, 671 kindergartners (mean age 4.6 years at the start of the study) were assessed on non-symbolic and symbolic comparison skills at six time points, and on their basic math achievement (divided into math fluency and math reasoning), and mapping skills at the end of first grade. Multivariate latent growth curve models show an interrelation between (the development of) non-symbolic and symbolic comparison skills. Results furthermore reveal symbolic comparison skills as the most important predictor of mapping skills and basic math achievement. Growth in non-symbolic comparison skills predicted math fluency in first grade.

By the age of five, general numerical awareness has started to develop; this leads to individual differences in children's numerical development (e.g., Aunio & Niemivirta, 2010; Stock, Desoete, & Roeyers, 2010; Toll, Van der Ven, Kroesbergen, & Van Luit, 2011). An important aspect of early numerical development is learning the connection between number symbols and their corresponding quantities ('4' matches a set of four objects). These skills are referred to as mapping skills (e.g., Kolkman, Kroesbergen, & Leseman, 2013), and are, like in the present study, frequently measured with a number line task in which children need to estimate the position of a given number on a (horizontal) number line. During a number line task children are challenged to map number words and symbols to their corresponding magnitude (e.g., Booth & Siegler, 2006). Another manner to measure mapping skills is a two-alternative forced-choice task. In this task, children are shown a target representation of one quantity (symbolic or nonsymbolic) and are required to choose which of the two alternative representations (nonsymbolic or symbolic) matches the target (Mundy & Gilmore, 2009). Mapping skills become more accurate (i.e., linear) during development (Booth & Siegler, 2006; Siegler & Booth, 2003) and were found to be important predictors for basic math performance (Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013; Kolkman, Kroesbergen, & Leseman, 2013). Basic math performance of children aged five to eight years old can roughly be divided into two different skills: fluently memorizing basic math facts (e.g., $4 + 3 = 7$), often measured with a time-pressured task, and more mathematical problem solving in which the context is described and basic reasoning skills are evoked (e.g., seven people are in the bus, two people get out at the next stop; how many people are in the bus after the bus stop?). In the present study those two skills are called math fluency and math reasoning. Mapping skills have found to be important for both skills (for speeded arithmetic, see for example Brankaer, Ghesquière, & De Smedt, 2014; for mathematical problem solving, see for example Kolkman, Kroesbergen, & Leseman, 2013). Little is known, however, about numerical precursors of mapping skills (Friso-van den Bos, Kolkman, Kroesbergen, & Leseman, 2014) and the basic math performance of children. Hence, the overall goal of the present study is to gain more insight in this underlying developmental path.

A common idea in the existing literature is that the ability to discriminate between magnitudes, both non-numerical, e.g. dots, and numerical, is an important prerequisite for mapping and other basic mathematical skills (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013;). It is assumed that both *non-symbolic* comparison skills (i.e., discriminating between dots, cubes or sticks) and *symbolic* number comparison skills (i.e., discriminating between Arabic symbols; below referred to as symbolic comparison skills) are important for the mapping of number symbols to non-symbolic quantities (e.g. Gilmore, McCarthy, & Spelke, 2010; Jordan, Glutting, & Ramineni, 2010). This distinction between

* Corresponding author. Tel.: +31 30 253 4614; fax: +31 30 253 2352.
E-mail addresses: S.W.M.Toll@uu.nl (S.W.M. Toll), S.VanViersen@uva.nl (S. Van Viersen), E.H.Kroesbergen@uu.nl (E.H. Kroesbergen), J.E.H.VanLuit@uu.nl (J.E.H. Van Luit).

http://dx.doi.org/10.1016/j.lindif.2014.12.006
1041-6080/

non-symbolic and symbolic comparison skills is based on the triple code model of Dehaene (1992; 2001). According to this model, there are three different types of representations for numbers (so-called codes): the analogue magnitude code (semantic knowledge about the proximity and relative size of quantities), the auditory verbal code (the ability to enumerate the counting row), and the visual code (Arabic representations). Dehaene (2001) hypothesized that all children are born with a system for non-symbolic quantity representation and that this can be measured as early as the first months of life. This suggests that children have an innate understanding of non-symbolic magnitudes (Dehaene, 1992; Feigenson, Dehaene, & Spelke, 2004) and is confirmed by studies showing that preschoolers are capable of comparing and adding large sets of elements without counting (e.g., Barth, La Mont, Lipton, & Spelke, 2005). During early childhood, symbolic knowledge develops based on increasing experience with number words (verbal code) and number symbols (visual code). The ability to translate between those Arabic numerals and verbal number words places critical constraints on young children's math development (Göbel, Watson, Lervåg, & Hulme, 2014). The connection between the three codes and their relation in math development has been studied extensively during the last decade (e.g., Sarama & Clements, 2009).

Besides Dehaene's model (1992; 2001), other models exist that make a distinction between non-symbolic and symbolic comparison skills. Von Aster and Shalev (2007), for example, propose a four-step developmental model with a core system of magnitude (i.e., cardinality) at the first step, the verbal number system at the second step, the Arabic number system at the third step, and a mental number line (i.e., ordinality) as a final step. Within this model, the non-symbolic comparison skills can be found in the first step, whereas the symbolic comparison skills are represented in the second and third steps. Integrating those non-symbolic and symbolic comparison skills directly relates to the ability to approximate calculations or create a spatial image of ordinate numbers (e.g., Siegler & Opfer, 2003). Recently, Geary (2013) distinguished between mechanisms that facilitate children's early numeracy learning. These mechanisms may include an inherent sense of magnitude (i.e., non-symbolic comparison skills), fluent mapping of basic mathematical symbols onto this intuitive number sense, and the ability to explicitly operate on these symbols and understand the logical relations among them (i.e., symbolic comparison skills).

In addition to theoretical frameworks, empirical evidence is available on the existence of non-symbolic and symbolic comparison skills as key aspects of numerical development. However, based on this evidence, it can be concluded that the relation between non-symbolic and symbolic comparison skills remains unclear. Yet, there exists a presumption that non-symbolic comparison skills and symbolic comparison skills are separable but dissociable components; they do not share the same underlying ability (Kolkman, Kroesbergen, & Leseman, 2013). The results of Friso-van den Bos, Kroesbergen, and Van Luit (2014) also support the idea that non-symbolic and symbolic number processing are distinguishable processes at kindergarten age. From the growing body of research on this topic it is furthermore known that both functions follow different developmental trajectories during childhood (Kolkman, Kroesbergen, & Leseman, 2013). Clear evidence about how these two skills are related is nevertheless lacking. Whereas some studies found significant correlations across performance on non-symbolic and symbolic comparison tasks (e.g., Gilmore, Attridge, De Smedt, & Inglis, 2014) or reveal a reciprocal nature of the relation between the non-symbolic and symbolic comparison skills (Gilmore et al., 2010), other studies illustrate the mastering of non-symbolic (comparison) skills as precondition for developing symbolic (comparison) skills (Kolkman, Kroesbergen, & Leseman, 2013). Yet, there are results that reveal no predictive association between non-symbolic number comparison and symbolic comparison six months later (Sasanguie, Defever, Maertens, & Reynvoet, 2014).

Whether non-symbolic or symbolic comparison skills are more important for early numeracy and math development is still a subject of discussion as well. Recently, De Smedt et al. (2013) reviewed neurocognitive and behavioral studies that tested the association between numerical processing and mathematics achievement and conclude that results are consistent across studies for the symbolic comparison skills, but rather contradictory for the non-symbolic comparison skills. Whereas some studies hypothesize non-symbolic understandings of magnitude as a necessary precondition for learning to associate a perceived number of objects with symbolic number words or number symbols (Von Aster, Schweiter, & Zulauf, 2007), other authors believe that the symbolic numerical skills of children might be more important than the previously developed quantitative abilities (e.g., Bartelet, Vaessen, Blomert, & Ansari, 2014). Predictive relationships were found between non-symbolic comparison skills and basic math performance in five to eight-year-old children (Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Gilmore et al., 2010; Inglis, Attridge, Batchelor, & Gilmore, 2011) as well as between symbolic comparison skills and basic math performance in six to eight-year-old children (e.g., Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012). This last group of studies states that non-symbolic comparison skills play a subordinate role in learning math in contrast to the important role of symbolic comparison skills (LeFevre et al., 2010), since an effect of non-symbolic comparison skills on math performance was mediated by symbolic comparison skills (Holloway & Ansari, 2009; Xenidou-Dervou, De Smedt, Van der Schoot, & Van Lieshout, 2013). Results of a longitudinal study in which different measures of math achievement were used (both a timed arithmetic test and a general curriculum-based math test; as will be the case in the present study), also emphasize the dominant role of learning experiences with symbols for later math abilities (Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Moreover, De Smedt et al. (2013) put forth the explanation that the kinds of representations and processes measured by the non-symbolic comparison tasks may be unimportant for school-relevant math skills. But even though no consensus has been reached about the influence of non-symbolic and symbolic comparison skills, most researchers agree that, eventually, the symbolic comparison skills are gradually integrated with existing nonverbal knowledge, resulting in more complex cognitive representations in which number symbols and words are connected to quantity representations (i.e., non-symbolic comparison skills) and that these specific skills can be referred to as mapping skills (Dehaene, 2001; Krajewski & Schneider, 2009; Mundy & Gilmore, 2009; Mussolin, Mejias, & Noël, 2010).

Since longitudinal evidence for the contribution of non-symbolic or symbolic comparison skills is scarce (De Smedt et al., 2013), the present study aims to provide additional grounds for this topic. In order to achieve this overall goal, there are two research goals. The first goal is to examine the nature of the developmental relationship between non-symbolic and symbolic comparison skills throughout kindergarten over a period of two and a half years. Based on previous longitudinal studies in this age group (e.g., Gilmore et al., 2010, 2014), it is hypothesized that a mutual relationship exists between non-symbolic comparison skills and symbolic comparison skills. The second goal is to examine the contribution of both non-symbolic and symbolic comparison skills in predicting mapping and other basic mathematical skills. In other words, it was tested whether it is possible to predict math performance at the end of first grade, after approximately one school year (i.e., ten months of education) of formal math instruction, based on children's numerical (non-symbolic and symbolic) comparison skills and their growth in those comparison skills throughout kindergarten. Because of the enormous variation in which mathematical performance in the early school years is measured, three ways to measure basic mathematical skills, including the mapping of number symbols to non-symbolic quantities (i.e., mapping skills), fluently memorizing basic math facts (i.e., math fluency) and mathematical problem solving (i.e., math reasoning), are distinguished in the present study. By doing so, we aim to provide more insight in the specific roles of (non-)symbolic comparison skills for learning mathematics.