Learning and Individual Differences 38 (2015) 61–67

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

A specific misconception of the equal sign acts as a barrier to children's learning of early algebra ☆

Caroline E. Byrd *, Nicole M. McNeil **, Dana L. Chesney [1], Percival G. Matthews [2]

University of Notre Dame, United States

ARTICLE INFO

Article history:
Received 2 January 2014
Received in revised form 8 December 2014
Accepted 1 January 2015

Keywords:
Conceptual development
Mathematics
Individual differences
Learning
Early algebra

ABSTRACT

Children's equal sign understanding affects learning of early algebra. Most studies to date have focused exclusively on the presence of *relational* interpretations of the equal sign (e.g., "the same as" or "equal to"), without examining how different types of *non-relational* interpretations affect learning. Children's (3rd and 5th graders; *M* age = 9 yrs, 11 mos) equal sign interpretations were measured prior to instruction on mathematical equivalence. In addition to helpful effects of relational interpretations, we hypothesized that an arithmetic-specific interpretation (e.g., "what something adds to") would be more likely to hinder children's learning than would other non-relational interpretations. Results supported these hypotheses. Presence of relational interpretations was helpful in both grades, and an arithmetic-specific equal sign interpretation negatively predicted 5th graders' end-of-year early algebra performance. Equal sign interpretations were not associated with arithmetic performance in either grade. Results extend our understanding of how equal sign interpretations shape children's mathematics learning.

1. Background

Children's misconceptions can act as barriers to learning. In mathematics, children sometimes construct overly narrow knowledge based on their initial experiences with a topic and subsequently overgeneralize that knowledge to problems that overlap with—but do not map directly onto—their original experiences. Examples include multi-digit subtraction (Brown & VanLehn, 1988), decimal fractions (Resnick et al., 1989), exponential expressions (Cangelosi, Madrid, Cooper, Olson, & Hartter, 2013), and the additive inverse (McGowen & Tall, 2013). Once integrated into children's cognitive framework, overly narrow knowledge persists and interferes with subsequent learning. Indeed, children often need to "re-conceptualize deeply rooted misconceptions that interfere with their learning" before they can truly learn new concepts (NRC, 1999, p. 176). The present study examined children's narrow interpretation of the equal sign as a barrier to learning early algebra.

We focused on children's equal sign interpretation because it is widely regarded as important for success in algebra (e.g., Alibali, Knuth, Hattikudur, McNeil, & Stephens, 2007; Booth & Davenport, 2013; Carpenter, Franke, & Levi, 2003; Knuth, Stephens, McNeil, & Alibali, 2006; McNeil & Alibali, 2005a, b). The equal sign (=) is a relational symbol, indicating that the two sides of an equation are equal and interchangeable (Kieran, 1981); however, children tend to view the equal sign operationally, meaning "add up the numbers" or "the answer" (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; De Corte & Verschaffel, 1981; Kieran, 1981; Matthews & Rittle-Johnson, 2009; McNeil & Alibali, 2005a; Oksuz, 2007; Perry, 1991; Sáenz-Ludlow & Walgamuth, 1998). Although operational interpretations allow children to solve traditional arithmetic problems (e.g., $2 + 4 = \_\_$) correctly, they do not facilitate success with more complex equations. Indeed, "virtually all manipulations on equations require understanding that the equal sign represents a relation" (Carpenter et al., 2003, p. 22). Without a relational understanding, the algebraic principle of maintaining equality is nonsensical, and children are left struggling to memorize countless, seemingly arbitrary rules for transforming equations (Falkner, Levi, & Carpenter, 1999; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Seo & Ginsburg, 2003; Steinberg, Sleeman, & Ktorza, 1990).

Evidence suggests that children's understanding of the equal sign affects early algebra learning and performance. Knuth et al. (2006) found a strong positive correlation between middle school students' equal sign understanding and their performance solving equations, such as $4m + 10 = 70$. Regardless of the strategy used to solve the equations (algebraic or arithmetic), students who interpreted the equal

☆ Thanks to Heather Brletic-Shipley and April Dunwiddie for their input and help with coding. This research would not have been possible without the support of our participating administrators, teachers, parents, and students.

* Correspondence to: C.E. Byrd, Department of Psychology, 118 Haggar Hall, University of Notre Dame, Notre Dame, IN 46556, United States. Tel.: +1 574 631 0963.
** Correspondence to: N.M. McNeil, Department of Psychology, 118 Haggar Hall, University of Notre Dame, Notre Dame, IN 46556, United States. Tel.: +1 574 631 5678.
E-mail addresses: cbyrd@nd.edu (C.E. Byrd), nmcneil@nd.edu (N.M. McNeil).
[1] Dana L. Chesney is now at Ohio State University.
[2] Percival G. Matthews is now at University of Wisconsin–Madison.

http://dx.doi.org/10.1016/j.lindif.2015.01.001
1041-6080/

sign relationally were more likely than students who did not to solve the equations correctly. This association held even when controlling for standardized mathematics test scores. This suggests that equal sign understanding matters for equation-solving success. However, the study was cross-sectional, so it could not be determined if relational understanding preceded equation-solving success, or vice versa.

Alibali et al. (2007) addressed this limitation by measuring changes in equal sign understanding and performance solving equations at four time points (fall of 6th and 7th grades and fall and spring of 8th). They focused on equivalent equations problems—problems that require children to determine whether the value of a variable (e.g., n) is the same in two equations (e.g., $2 \times n + 15 = 31$ and $2 \times n + 15 - 9 = 31 - 9$). Results suggested that relational understanding both precedes and predicts advanced solving. Students who interpreted the equal sign relationally were more likely than those who did not to recognize the equivalence of the two equations, and relational understanding tended to precede this recognition. Moreover, the earlier students acquired relational understanding, the better their performance at the end of 8th grade, suggesting that children's equal sign understanding can affect subsequent early algebra performance.

Although the aforementioned studies suggest that children's equal sign interpretations can shape how they learn early algebra, both studies had limitations. Both examined performance using only one particular type of early algebra problem, so neither established that relational understanding of the equal sign affects children's understanding of early algebra more broadly. More importantly, neither study considered that the *type* of non-relational interpretation could be important. Thus, it is unclear whether findings are attributable to the presence of a relational interpretation, or to the absence of a particular kind of non-relational interpretation.

Certain non-relational interpretations may be more counterproductive than others for early algebra learning. Identification of misconceptions that hinder learning would be a potentially powerful tool for remediation of children who struggle with early algebra. Unfortunately, it is unclear if a specific non-relational way of interpreting the equal sign hinders learning because the majority of studies have classified children's interpretations as only relational versus non-relational.

A few studies have differentiated among various non-relational interpretations (e.g., Jones, Inglis, & Gilmore, 2011; McNeil & Alibali, 2005a, b). McNeil and Alibali (2005b) suggested that defining the equal sign using arithmetic-specific words and phrases such as "add, subtract, or sum"[3] is less helpful for future learning than defining it using more general words such as "end, answer, or result." Supporting this hypothesis, they found that children who made particular errors in encoding and solving equations and defined the equal sign using arithmetic-specific words were less likely to learn from an intervention on equations than peers who did not make those same errors. However, this study did not examine the contribution of children's equal sign definitions independent of encoding and solving. Moreover, it only examined learning during one session and only assessed performance on one type of equation.

In the present study, we tested the hypothesis that an arithmetic-specific view of the equal sign confers specific risk for poor learning of early algebra over the course of a school year. We measured individual differences in children's equal sign interpretations at the start of the school year to examine if those differences could predict performance on a multi-item measure of early algebra understanding at the end of the school year. In the intervening months, children were taught how to solve *mathematical equivalence problems*—arithmetic problems with operations on both sides of the equal sign (e.g., $3 + 4 + 5 = 3 + \_\_$, Perry, Church, & Goldin-Meadow, 1988). We hypothesized that children who provided relational definitions of the equal sign at the start of the school year would perform better on early algebra items at year's end than those who gave non-relational definitions. Moreover, we hypothesized that children who provided arithmetic-specific definitions (e.g., "add up the numbers") at the outset would perform worse on the early algebra items than children who provided other non-relational definitions (e.g., "the answer"). Additionally, because an arithmetic-specific view is compatible with typical arithmetic, we hypothesized that the negative effects of an arithmetic-specific view would not extend to traditional arithmetic items.

2. Method

2.1. Participants

One hundred fourteen children entered the study at the start of the school year. Fourteen did not complete the end-of-year assessment because they were absent or had moved. Thus, the final sample included 100 children (45 boys, 55 girls; 27 third graders, 73 fifth graders). Children ranged in age from 7 years 10 months to 11 years 8 months ($M = 9$ years 11 months). Children had not previously received any explicit instruction on mathematical equivalence or early algebra before the study. During the school year, children received teacher-designed instruction from teachers in a school-initiated collaborative working group whose members chose to focus on improving children's understanding of mathematical equivalence. The collaborative working group was part of a school initiative formed independently of the researchers. Teachers chose their own working groups, so researchers had no control over the included grade levels in the group. The target working group was led by a teacher with a long history of working with researchers to promote children's understanding of mathematical equivalence, so she encouraged her group to focus on that issue.

The study was conducted at a public school in a southeastern U.S. exurb that used the Harcourt Math (Malestsky & Andrews, 2004) curriculum. The school's percentage of students scoring at or above grade level on the state end-of-year mathematics achievement test was 80.6% for third graders and 79.7% for fifth graders. The state average was 73.2% for third graders and 69.6% for fifth graders. The school's racial/ethnic makeup was 69% white, 24% black, 6% Latino, and 1% Asian. Approximately 37% of children received free or reduced-price lunch.

2.2. Design

At the start of the school year, children completed a measure assessing their equal sign interpretation and performance solving mathematical equivalence problems. Teachers were unaware of our hypotheses and they were not specifically focused on assessing differences in children's relational or non-relational equal sign interpretations. Throughout the school year, children received teacher-designed instruction on how to solve mathematical equivalence problems and associated problems, such as solving for unknown values represented by variables, as a supplement to their regular instruction. Teachers fully collaborated on these lessons, with the primary goal of teaching children to solve mathematical equivalence problems correctly. Teachers met regularly to set goals, plan activities, and discuss children's progress. Sometimes children across classrooms were brought together for team-taught lessons. Other times a single teacher gave a lesson she developed to multiple classrooms. At the end of the school year, children completed a written assessment measuring performance on early algebra and challenging arithmetic problems (see Tables 1 and 2).

[3] McNeil and Alibali (2005b) referred to equal sign definitions that used arithmetic specific words or phrases as "arithmetic operator" definitions. However, given the amount of overlap between this term and the more general and more widely known "operational" definition, we will instead refer to them as "arithmetic-specific" definitions hereafter to clarify that they are a specific form of the broader operational view.