Learning and Individual Differences 41 (2015) 21–29

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

Examining factor structures on the Test of Early Mathematics Ability — 3: A longitudinal approach

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ARTICLE INFO

Article history:
Received 22 March 2014
Received in revised form 24 June 2015
Accepted 24 June 2015

Keywords:
Bayesian method
Early childhood mathematics
Bi-factor model
Longitudinal data
Test of Early Mathematics Ability — 3

ABSTRACT

Assessment of early mathematics skills in young children is important both psychometrically and practically. The TEMA-3 (Ginsburg & Baroody, 2003) yields a total score (Math Ability Score) and provides useful information on children's overall mathematics performance, but not about performance in specific skill areas that are targeted by state early learning guidelines and by the National Council of Teachers of Mathematics (NCTM) and Common Core State Standards for Mathematics as important content for Understanding Number in early childhood. This study examined the factor structure of the TEMA-3 to examine whether items reflect the conceptual categories in the TEMA-3 Examiner's Manual (Ginsburg & Baroody, 2003) or a priori categories reflecting Understanding Number. Longitudinal data from 389 children (182 males), mean age 54.46 months at first assessment, were obtained in fall and spring of pre-kindergarten, spring of kindergarten, and spring of 1st grade. Changes in factor structure are examined across measurement points. The a priori factor structure was found by confirmatory factor analysis to better fit the data. The numbers of factors identified increased across time. Subscale scores reflecting children's knowledge of specific mathematics concepts related to number concepts could enable teachers and parents to select activities to strengthen children's mathematical knowledge in those skill areas.

1. Introduction

Knowledge of mathematics gained during the pre-kindergarten period is recognized as a critical component of children's mathematics learning at school age (Brenneman, Stevenson-Boyd, & Frede, 2009; Denton & West, 2002; Ginsburg, Lee, & Boyd, 2008; National Mathematics Advisory Panel, 2008; Whitehurst, 2001). The National Mathematics Advisory Panel's final report (NMP, 2008) notes

> "Most children acquire considerable knowledge of numbers and other aspects of mathematics before they enter kindergarten. This is important because the mathematical knowledge that kindergartners bring to school is related to their learning for years thereafter". [NMP (2008) p. xvii]

The joint report from the National Association for the Education of Young Children and National Council of Teachers of Mathematics (2002) identifies the importance of research-based, high-quality, challenging, and accessible mathematics instruction in pre-school to give children a strong start in the conceptual and procedural understandings of mathematics. The National Research Council Committee on Early Childhood Mathematics (2009) identifies core areas of number, geometry, spatial relations and measurement as foundational mathematics content for children aged 3 through 6 years. Similarly the National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points (2006) identifies number and operations, geometry, and measurement as critical areas of instruction and learning in pre-kindergarten. More recently, the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) identified skills for kindergarten children targeting understanding of whole numbers. This study focuses on the knowledge and process skills that characterized *Understanding Number*, a content area found across these publications.

In Table 1 are the number concepts and process skills reflecting *Understanding Number* that we identified in the NCTM Principles and Standards (2000), National Council of Teachers of Mathematics (2006), NCTM Essential Understanding of Number & Numeration (2010), the National Research Council Committee on Early Childhood Mathematics (2009) and National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). It is apparent from the table that there is considerable but not complete

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http://dx.doi.org/10.1016/j.lindif.2015.06.003
1041-6080/

Table 1
Mathematics knowledge and process skills: Understanding number.

NCTM principles and standards	NCTM curriculum focal points	NCTM essential understanding of number and numeration	Early childhood mathematics NRC Committee 2009	CCSSM
Count with understanding and recognize "how many" in sets of objects	Count to determine number amounts and compare quantities	Counting includes cardinality and ordinality of sets of objects	Using the number word list in counting and counting to find the cardinality of a set	Count to tell the number of objects
Connect number words to the quantities they represent	Understand that number words refer to quantity	Patterns in the number–word sequence provide a foundation for the abstract number concept	Giving a number word for the numerosity of a set	Count verbally
Be able to represent and compare whole numbers	Compare and order whole numbers	Using numbers to describe relationships between or among quantities depends on identifying a unit	Comparing two quantities to find out how much more or how much less one is than the other	Compare numbers
Understand various meanings of addition and subtraction, multiplication and division	Develop strategies for adding and subtracting whole numbers and understand the meaning of multiplication and division through use of representations	Not referenced	Put together two sets to make a total: take apart a number to make two addends	Represent and solve addition, subtraction, multiplication and division problems
Develop fluency with basic number computations for addition and subtraction, multiplication and division	Develop fluency with efficient procedures	Not referenced	Not referenced	Compute fluency
Understand the relative position and magnitude of whole numbers and of ordinal and cardinal numbers	Understand the sequential order of the counting numbers and their relative magnitudes	Number–word sequence, combined with the order inherent in the natural numbers can be used in the foundation of counting	Knowing how to say the sequence of number words	Know number names and the count sequence
Develop understanding of place value and the base-ten number system	Develop an understanding of the base-ten numeration and place value	Our base-ten number system allows forming a new place-value unit by grouping ten of the previous place-value units	Not referenced	Understand place value

Note: Common Core State Standards for Mathematics (CCSSM); National Council of Teachers of Mathematics (NCTM); and National Research Council (NRC).

agreement among these publications on *Understanding Number*. However, assessment of these early concepts and skills is a different story. Although approaches to formative assessments of early mathematics knowledge have been described (e.g., Bowman, Donovan, & Burns, 2001; Lee, Lembke, Moore, Ginsbura, & Pappas, 2007; National Association for the Education of Young Children & National Council of Teachers of Mathematics, 2002), there are few summative assessments in mathematics appropriate for young children. However, one summative assessment is the Test of Early Mathematics Ability — 3 (TEMA-3: Ginsburg & Baroody, 2003). The TEMA-3 is designed to assess mathematics understanding and skills of young children and consists of 72 items targeting informal and formal mathematics knowledge. The Informal conceptual categories include items reflecting numbering (e.g., "*How many cats do you see.*"), number comparison (e.g., "*Which side has more?*"), calculation (e.g., "*How many does he have altogether?*"), and concepts (e.g., "*How many stars did you count?*"). The Formal conceptual categories include items reflecting numeral literacy (e.g., "*What number is this?*"), number facts (e.g., "*How much is two take away one?*"), calculation (e.g., "*How many points does he have altogether*?"), and concepts (e.g., "*Which number sentences here are correct for this word problem*?"). In Table 2a the conceptual categories are represented by sets of items on the TEMA-3.

TEMA-3 assessments yield a standardized total score, called a Math Ability Score (MAS). These total scores yield useful information on children's overall performance in mathematics, and MASs have been used in studies as criterion outcome measures (e.g., Huntsinger, Jose, Liaw, & Ching, 1997; Molfese, Yaple, Helwig, Harris, & Connell, 1992; Zhou, Cheng, Mottram & Rosenblum, 1999). However, it is unknown whether TEMA-3 items that might be grouped into the conceptual categories described in the TEMA-3 Examiner's Manual (Ginsburg & Baroody, 2003) are a good statistical fit to those categories; category scores have not been validated. Thus, researchers and educators interested in children's knowledge of specific mathematics concepts cannot use the TEMA-3 MAS scores to determine this information.

Table 2a
Item clusters corresponding to the "categorical" factor structure.

Categories		Items
Informal mathematics	Numbering	1, 2, 3, 5, 6, 10, 11, 12, 13, 21, 22, 23, 24, 28, 31, 33, 36, 38, 39, 40, 42, 48, 68
	Number comparisons	4, 19, 20, 27, 37, 58
	Calculation	8, 16, 26, 32, 65, 67, 72
	Concepts	7, 9, 17, 25
Formal mathematics	Numeral literacy	14, 15, 29, 30, 35, 44, 45, 60
	Number facts	41, 43, 46, 50, 51, 54, 56, 61, 70
	Calculation	49, 52, 55, 57, 59, 62, 63, 64, 69, 71
	Concepts	18, 34, 47, 53, 66

The purpose of the present study is two-fold. The first purpose is to examine the subscale scores composed of items based on the conceptual categories described in the TEMA-3 Examiner's Manual (Ginsburg & Baroody, 2003). There are no published reports of the factor structure of the TEMA-3 available, so the present study investigated whether a factor analysis of item level data from the TEMA-3 would show a good fit with the four hypothesized Informal Mathematics categories and the four hypothesized Formal Mathematics categories described in the Examiner's Manual (page 34). The eight TEMA-3 "*categorical* factors" and the items associated with each factor are in Table 2a. Alternatively, a different factor structure might be a better fit to item level data. Based on our review of the concepts and process skills characterizing *Understanding Number* shown in Table 1 and the face validity of each TEMA-3 item including the accompanying instructions ("stimulus") given to the child for each item, seven a priori categories were created by the authors. Those categories are: counting objects (count to tell the number of objects), verbal counting (count verbally), numerical comparison (compare numbers), set construction (represent composition and decomposition of sets with objects, drawings, expressions or equations), numeral literacy (recognize or write whole number numerals), number facts (demonstrate fluency with basic number computations), and calculation (represent and solve addition, subtraction and multiplication problems). The seven a priori categories and related items are shown in Table 2b.[1] By comparing

[1] The original placement of nine items (3, 8, 12, 16, 26, 34, 52, 55, and 59) was changed based on a reviewer's comments and justification for placing these items into different a priori categories.