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Individual differences in kindergarten math achievement: The integrative roles of approximation skills and working memory

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ABSTRACT

Kindergarteners can conduct basic computations with large nonsymbolic (e.g. dots, objects) and symbolic (i.e. Arabic numbers) numerosities in an approximate manner. These abilities are related to individual differences in mathematics achievement. At the same time, these individual differences are also determined by Working Memory (WM). The interrelationship between approximation, WM and math achievement has been largely unexplored. Also, the differential role of nonsymbolic and symbolic approximation in explaining math competencies is yet unclear. We examined an integrative theoretical model on the association between approximation (addition and comparison) and WM in 444 kindergarteners. As expected, approximation entailed two distinct abilities (nonsymbolic and symbolic). Both abilities correlated with mathematics achievement (i.e. counting and exact arithmetic), even when WM was taken into account. The association between nonsymbolic approximation and math achievement was completely mediated by symbolic approximation skills. These findings add to our understanding of the cognitive architecture underlying kindergarten math achievement.

1. Introduction

From the girl, who realizes she was given less candies than her older sister, to the adult at the counter, who estimates his change, math is everywhere. There are large differences between individuals in the way they develop mathematical competencies (Geary, 2011). Research so far has been very fruitful in identifying math-specific (neuro)cognitive precursors for mathematical achievement at the early stages of development (see Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). Crucial roles have been attributed to children's abilities to conduct simple computations in an approximate manner with large nonsymbolic (e.g. objects, dots, sounds), and symbolic (i.e., Arabic numbers) numerosities (e.g. Barth et al., 2006; Gilmore, McCarthy, & Spelke, 2007, 2010; Libertus, Feigenson, & Halberda, 2011). However, the differential role that nonsymbolic and symbolic approximate magnitude skills play in the early development of children's math proficiency and deficiency is an ongoing discussion in the literature (e.g. Gilmore et al., 2010; Holloway & Ansari, 2009; Mundy & Gilmore, 2009; Noël & Rouselle, 2011).

Previous research in the domain has mostly been conducted with small populations limiting the generalizability of their results. Also, few have focused on the kindergarten age, i.e. before the start of formal primary education, or have used multiple measures of approximation skills. Finally, the effect of the domain-general capacity of Working Memory (WM), which has been demonstrated to affect approximation processing (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012; Xenidou-Dervou, van Lieshout, & van der Schoot, in press) has not yet been accounted for in related research.

The present study tried to address these issues by investigating how nonsymbolic and symbolic approximation skills were related to kindergarten math achievement when taking into account WM capacity. We conducted a large-scale correlational study that allowed us to examine the factorial structure and interrelation of the aforementioned cognitive skills. With this design, we were able to address the issue of which specific role nonsymbolic and symbolic approximation skills (addition and comparison) could play in explaining individual differences in math achievement at the kindergarten age.

1.1. Nonsymbolic and symbolic approximation skills

Learning addition in the form of $a + b = c$, is a strenuous process and its mastery may take years within primary school instruction (Ashcraft & Fierman, 1982; Hamann & Ashcraft, 1985). Nonetheless, kindergarteners appear to perform easily above chance level in addition tasks that call for an approximate response (Barth et al., 2006; Gilmore et al., 2007, 2010). The literature assumes the existence of an evolutionary ancient system, the so-called Approximate Number System (ANS), with which humans and animals are enriched in order to estimate stimuli in nature (see Brannon, Jordan, & Jones, 2010; Feigenson et al., 2004; Piazza, 2010). Preschool children, before having acquired formal school instruction, have been consistently demonstrated to be able to

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successfully compare and add approximately large quantities of nonsymbolic stimuli (Barth et al., 2006; Gilmore et al., 2010; Libertus et al., 2011; Mazzocco, Feigenson, & Halberda, 2011; Xenidou-Dervou et al., in press). They can do so even when these quantities are presented in different modalities and formats (Barth, Beckmann, & Spelke, 2008; McNeil, Wagner Fuhs, Keultjes, & Gibson, 2011). Performance in tasks assessing these skills is characterized by the so-called ratio effect. The more the ratio difference between two target numerosities approaches 1, the harder it is to compare them (Barth et al., 2006; Izard & Dehaene, 2008). Additionally, research has shown that nonsymbolic approximate addition is as accurate as nonsymbolic approximate comparison, which calls only for the comparison of two numerosities (Barth et al., 2006). The aforementioned skills are referred to here as *nonsymbolic approximation skills*.

Five-year old children are able to solve corresponding approximate comparison and addition problems even with symbolic stimuli when they have not actually been formally taught large-numerosity symbolic arithmetic (Gilmore et al., 2007; McNeil et al., 2011). These abilities are referred to as *symbolic approximation skills* and they are subject to the same characteristic effects as nonsymbolic approximation (Gilmore et al., 2007). This shared cognitive profile suggests that children recruit their ANS representations when engaging in symbolic approximation. In other words, it is assumed that symbolic approximate representations are mapped onto and are fostered by one's preexisting nonsymbolic representations. More precise mapping representations between nonsymbolic and symbolic numerosities have been associated with better mathematical achievement in young children (Mundy & Gilmore, 2009). In this respect, the approximation system is sometimes viewed as a single system.

The precise role that nonsymbolic approximation plays in the process of acquiring formal mathematical knowledge and skills is yet unclear. Some assume that it comprises the foundation for learning symbolic arithmetic, i.e. as taught in school (Barth et al., 2006; Gilmore et al., 2007; Halberda & Feigenson, 2008; Halberda, Mazzocco, & Feigenson, 2008; Libertus et al., 2011; Mazzocco et al., 2011; Mundy & Gilmore, 2009). In line with this view, Gilmore et al. (2010) found preschoolers' nonsymbolic approximation skills to be predictive of their counting and mathematical skills. This occurred irrespective of the children's verbal intelligence and reading skills.

Others, however, support that nonsymbolic approximation skills are not determinant for early mathematics achievement (Le Corre & Carey, 2007; Noël & Rouselle, 2011). It is assumed that the skills in question become important only after the age of eight years (Noël & Rouselle, 2011). This assumption is supported by findings, where nonsymbolic task performance failed to account for individual differences in early age math achievement (e.g. De Smedt & Gilmore, 2011; Holloway & Ansari, 2009; Mundy & Gilmore, 2009). Instead, nonsymbolic abilities played an important role for children of older ages (Mazzocco et al., 2011; Piazza et al., 2010). As outlined by Noël and Rouselle (2011), the age factor is crucial. The aforementioned studies comprised children from six years of age and older. The critical age of kindergarten, however, has received less attention. Also, few studies have made use of multiple approximation measures. Sasanguie, De Smedt, Defever, and Reynvoet's (2011) cross-sectional study tested kindergarteners in nonsymbolic and symbolic comparison and number line tasks. They found a strong association between symbolic but not nonsymbolic magnitude comparison with kindergarteners' math achievement. The small population sample used in this study, however, restricts the generalizability of the results. Furthermore, one may wonder as to how validly number line tasks measure numerical representation because: "Number line estimation tasks assess only one aspect of children's numerical representations, namely, the linearity of children's symbolic representations. Tasks involving more general nonsymbolic representations are necessary to fully investigate the role of this system in mathematics learning." (Mundy & Gilmore, 2009; pp. 492). Inspired by Barth, La Mont, Lipton, and Spelke (2005), Barth et al. (2006) and Gilmore et al. (2007) measures, the present correlational study assessed nonsymbolic and symbolic approximation skills in comparison as well as in addition, in a large population sample of kindergarteners. This design permitted the usage of structural equation modeling techniques that could determine the latent structure and interrelation of the targeted cognitive systems; namely, approximation skills, WM and math achievement.

1.2. The role of WM

WM refers to the cognitive system that is dedicated to the short-term storage, regulation and manipulation of information in an online manner (Baddeley, 2003). Baddeley's multicomponent model of WM (Baddeley, 1986, 2003) incorporates a master system, the central executive (CE), which controls, monitors and regulates the processes of two auxiliary systems; the phonological loop (PL) and the visuospatial sketchpad (VSSP). These "slave" subsystems are responsible for the temporary storage of phonological and visuospatial information respectively. All WM components have been shown to play a role in math performance. The extent of involvement of each component is dependent upon the different requirements of a given task (Noël, 2009; Rasmussen & Bisanz, 2005; Simmons, Willis, & Adams, 2012; for a review see DeStefano & LeFevre, 2004).

In general, there is compelling evidence surrounding the importance of WM in explaining individual differences in mathematical achievement (Bull, Espy, & Wiebe, 2008; De Smedt, Janssen, et al., 2009; DeStefano & LeFevre, 2004; LeFevre, DeStefano, Coleman, & Shanahan, 2005; Passolunghi, Vercelloni, & Schadee, 2007; Raghubar, Barnes, & Hecht, 2010). More specifically, it has been strongly related to both counting (Geary, Hoard, Byrd-Craven, & DeSoto, 2004) as well as mental arithmetic (for a review see DeStefano & LeFevre, 2004). Recent findings addressed the association between children's approximation skills and WM. With a dual-task study, Xenidou-Dervou et al. (in press) demonstrated that the CE is necessary for *nonsymbolic* approximate addition processing in kindergarten. On the other hand, Caviola et al. (2012) showed that children's *symbolic* approximate addition called for PL or VSSP resources according to the demands and constraints of a task. Thus, various WM components are necessary for processing different approximation problems. In essence, these findings suggest that individual differences in approximation skills can be explained by individual differences in WM capacity. To our knowledge, however, the effect of this interrelationship on kindergarteners' math achievement has been unexplored.

1.3. The present study

In kindergarten, math achievement involves children learning how to count and beginning to understand the basic principles underlying addition (e.g. Geary, 2011). At this developmental stage counting skills form children's basis for learning how to conduct their first simple additions and constitute a measure of their math achievement (Geary, 2011; Geary et al., 2004). We addressed the question: How are kindergarteners' approximation and WM skills associated with these mathematical competencies? To the best of our knowledge, this study is the first to take into accumulating account the aforementioned skills in the process of math learning.

Amassing the findings presented in the previous sections, we formulated a comprehensive model on the integrative relationship of approximation, WM and math achievement in kindergarten age. As illustrated in Fig. 1, we hypothesized that: (1) Nonsymbolic and symbolic approximation skills would comprise two distinct abilities (Holloway & Ansari, 2009); (2) WM, as a domain-general cognitive ability, would predict math achievement beyond the effect of the domain-specific cognitive abilities (DeStefano & LeFevre, 2004; Geary, 2011; LeFevre et al., 2005; Raghubar et al., 2010); (3) WM would influence performance in both nonsymbolic (Xenidou-Dervou et al., in press) and symbolic approximation (Caviola et al., 2012); (4) Nonsymbolic approximation would