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Measuring the approximate number system in children: Exploring the relationships among different tasks

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ABSTRACT

Research has demonstrated that children and adults have an *Approximate Number System* (ANS) which allows individuals to represent and manipulate the representations of the approximate number of items within a set. It has been suggested that individual differences in the precision of the ANS are related to individual differences in mathematics achievement. One difficulty with understanding the role of the ANS, however, is a lack of consistency across studies in tasks used to measure ANS performance. Researchers have used symbolic or nonsymbolic comparison and addition tasks with varying types and sizes of stimuli. Recent studies with adult participants have shown that performance on different ANS tasks is unrelated. Across two studies we demonstrate that, in contrast to adults, children's performance across different ANS tasks, such as symbolic and nonsymbolic comparison or approximate addition, is related. These findings suggest that there are differences across development in the extent to which performance on nonsymbolic and symbolic tasks reflects ANS precision.

1. Introduction

In our everyday life we are surrounded by numbers and quantities: from the numbers on a page, to the items in a shopping basket and the coins in our pocket. Life becomes easier if we can efficiently remember, compare and order these quantities. We can choose the most numerous punnet of fruit, select the correct number of coins and join the shortest queue at the checkout. Children also deal with information about quantities in their play activities well before they start formal education. They may share sweets with their friends, compare the dots on two dice or count the pieces of a puzzle. Given the ubiquitous nature of numerical and quantity information, theorists have sought to understand how we represent and process numbers and quantities. In particular, researchers have explored whether individuals differ in how efficiently they can store and use numerical and quantity information, and how these differences arise. In this paper we explore how children represent and process information about quantities and whether there are consistent individual differences in their ability to do so.

Recently, psychologists have proposed the existence of an *Approximate Number System* (ANS), which is a cognitive system that allows us to represent and manipulate information about numbers and quantities (Cordes, Gelman, Gallistel, & Whalen, 2001; Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004). An approximate representation of quantity is generated when we observe a set of items. According to the theory, quantity information is represented in an imprecise manner on a ‘mental number line’ where smaller quantities are represented more precisely than larger quantities. It has been proposed that quantity information about a set of n items will be represented somewhere on the mental number line according to a normal distribution with a mean of n and standard deviation wn (Barth et al., 2006). Here the parameter w is known as the Weber fraction and varies across individuals so that the precision of people's ANS representations also varies. Individuals with a smaller w will tend to represent quantities more precisely than individuals with a larger w.

It has been suggested that the ANS is a universal system shared by infants, children, adults and animals (e.g., Barth, La Mont, Lipton, & Spelke, 2005; Brannon, 2005; Dehaene, 1997; Pica, Lemer, Izard, & Dehaene, 2004; Xu, Spelke, & Goddard, 2005). ANS representations of quantity can be used to determine which of two sets of items has more elements, to order several sets of items according to the quantity or even to decide whether the sum of two sets is greater or less than a third set. In the lab this is typically explored by asking participants to compare dot arrays or sequences of tones. However, it is frequently argued that these abilities could serve a practical purpose in allowing an animal or human to determine, for example, which tree contained more fruit. Due to the imprecise nature of ANS representations, success in making comparisons of this sort will depend on the ratio between the two sets of items. It is more difficult to accurately determine which fruit tree, or array of dots, contains more elements if the ratio of the quantities

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is closer to one. The accuracy with which an individual can select the more numerous of two sets of items will thus depend both on the ratio between the sets and the size of the individual's *w*.

It has been suggested that ANS representations of quantity are also involved when individuals process symbolic numerical information, i.e. number words and Arabic digits. Several authors have proposed that when exact symbolic representations of quantity are learned, they become mapped onto the pre-existing ANS representations (Le Corre & Carey, 2007; Noël & Rousselle, 2011; Wagner & Johnson, 2011). The timing and manner of this mapping are still under debate and may occur before (e.g. Wagner & Johnson, 2011) or after (e.g. Le Corre & Carey, 2007) children learn the cardinal meaning of symbolic numbers, or sometime later (e.g. Noël & Rousselle, 2011) and may or may not remain stable through life (Lyons, Ansari, & Beilock, 2012). Nevertheless, there is evidence that by school age imprecise ANS representations are connected to symbolic representations and continue to influence performance on numerical tasks, even when they involve symbolic numerical representations. For example, participants show a ratio effect even when they are asked to choose the larger of two Arabic digits (e.g. Holloway & Ansari, 2009; Moyer & Landauer, 1967; Sekuler & Mierkiewicz, 1977). There is some evidence that as well as individual differences in the precision of the ANS representations themselves, there may be individual differences in the mapping between symbolic and nonsymbolic (ANS) representations. For example, Mundy and Gilmore (2009) found that individual differences in mapping skill accounted for variance in mathematical ability over and above the variance accounted for by nonsymbolic and symbolic comparison tasks.

Evidence for the ANS has come from studies showing that, within certain ratio limits, adults, children, infants and animals can identify the larger of two sets of items with above chance accuracy and show ratio effects on performance. For example, Barth et al. (2006) showed adult participants two dot arrays presented sequentially on the screen and asked them to select the more numerous. The number of dots in each array ranged from 9 to 63 and the arrays were presented too quickly for the participants to be able to count them. Participants' accuracy on this task was above chance levels, and varied according to the ratio between the quantities presented: participants were more accurate when the ratio between the quantities was 0.7 than when it was 0.8. Further studies have shown that 3 year-old children can also perform above chance on similar tasks and again show the characteristic ratio effect on performance (e.g., Libertus, Feigenson, & Halberda, 2011). Infants as young as 6 months old can also detect changes in quantity in a habituation paradigm (Xu et al., 2005).

Much of the research exploring the ANS, particularly with children, has focused on discovering the types of ANS tasks that children can solve. So, for example, studies have revealed that children can compare, order, add, subtract and possibly even multiply and divide nonsymbolic quantities (typically dot arrays) with above chance accuracy (e.g. Gilmore, McCarthy, & Spelke, 2007; McCrink & Spelke, 2010). As a result of this effort to explore the limits of the ANS, there has been little exploration of what factors contribute to individual differences in children's success on these tasks. However, the few studies to consider individual level analysis have revealed that there are wide variations in children's success with these problems. For example, McCrink and Spelke (2010) explored children's ability to perform nonsymbolic multiplication in a simple animated task involving dot arrays. Although group level analyses suggested that children were able to perform above chance levels on a task involving multiplication by 4, individual level analyses indicated that in fact only 9 out of 16 participants were performing above chance. A consideration of how children's ability to solve ANS tasks varies and the extent to which individual differences are consistent across different ANS tasks is therefore overdue (Gilmore, 2013).

One area in which there has been a focus on individual differences concerns exploration of the relationship between ANS performance and mathematics achievement. Given that symbolic representations of number are mapped onto ANS representations, it has been proposed that the ANS may play a role in learning and performing mathematics and therefore individual differences in performance on ANS tasks will be related to individual differences in performance on mathematics tests. Data from a number of studies have supported this hypothesis (e.g. Bonny & Lourenco, 2013; Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008, Libertus, Feigenson, & Halberda, 2013; Libertus, Odic, & Halberda, 2012; Libertus et al., 2011; Lourenco, Bonny, Fernandez, & Rao, 2012; Mazzocco, Feigenson, & Halberda, 2011) although other studies have failed to (e.g. Castronovo & Göbel, 2012; Holloway & Ansari, 2009; Iuculano, Tang, Hall, & Butterworth, 2008; Kolkman, Kroesbergen, & Leseman, 2013; Price, Palmer, Battista, & Ansari, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Sasanguie, Van den Bussche, & Reynvoet, 2012; Vanbinst, Ghesquière, & De Smedt, 2012). The presence of this relationship may depend on the age of the participants and the nature of the tasks and measures involved (e.g. Inglis, Attridge, Batchelor, & Gilmore, 2011; Mundy & Gilmore, 2009; see De Smedt, Noël, Gilmore, & Ansari, 2013 or Gilmore, 2013 for reviews).

Studies exploring this relationship have used a range of tasks to measure ANS performance (e.g. nonsymbolic or symbolic comparison, nonsymbolic addition) involving either small (1–9) or larger quantities (up to 70) and measures of mathematics performance have included standardized mathematics tests, curriculum-based measures or simple arithmetic tasks. For example, Halberda et al. (2008) found that ANS performance in adolescence was related to mathematics ability in childhood, Libertus et al. (2011) found that 2–6-year old children's performance on a nonsymbolic comparison task was related to performance on a standardized mathematics test; Gilmore, McCarthy, and Spelke (2010) found that 5–6-year old children's performance on a nonsymbolic addition task was related to performance on a school mathematics test; and Lyons and Beilock (2011) found that adults' performance on a nonsymbolic comparison task was related to their score on a mental arithmetic test. Alongside these studies, research on dyscalculia, a specific learning disorder with persistent difficulties in mathematics achievement, has shown that these children perform more poorly on nonsymbolic comparison tasks compared to typically developing controls (e.g., Piazza et al., 2010). However, several studies have failed to find a link between ANS performance and mathematics achievement, or found that this link is dependent on the age of participants, task or measure involved. For example, Inglis et al. (2011) found that nonsymbolic comparison performance was related to mathematics achievement in children aged 7—9 years but not for adult participants; Holloway and Ansari (2009) found that performance on a symbolic comparison task, but not a nonsymbolic comparison task was related to mathematics achievement in 7- to 9-year-old children; Rousselle and Noël (2007) and De Smedt and Gilmore (2011) observed that children with dyscalculia showed only impairments in symbolic but not nonsymbolic comparison tasks; Iuculano et al. (2008) found no relationships between nonsymbolic comparison, addition or subtraction and arithmetic skills; and Price et al. (2012) found that adults' performance on three different variants of a nonsymbolic comparison task was unrelated to standardized maths performance.

One difficulty with interpreting the findings from these sets of studies is the range of tasks and stimuli that have been used. It is currently unknown whether tasks involving different operations, stimuli or number ranges give equivalent measures of ANS performance. If the ability to compare Arabic numerals up to 9 is unrelated to the ability to add large collections of dots, then it is unsurprising that there are varying results when comparing performance on just one of these tasks with mathematics achievement. However, few studies to date have used multiple ANS tasks within the same experiment, especially with child participants.

Some evidence exists from studies with adult participants to suggest that performance on different versions of ANS tasks is not related.