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The dual processes hypothesis in mathematics performance: Beliefs, cognitive reflection, working memory and reasoning

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ABSTRACT

In this paper, using data provided by an empirical study of students in a high school science course, we discuss key variables in the interaction between System 1 (S1) (intuitive and unconscious processes) and System 2 (S2) (analytical and conscious processes) in mathematical reasoning. These variables are: beliefs about oneself and about mathematics; cognitive reflection understood as a self-regulatory skill; working memory; and the evaluation of the deductive and probabilistic reasoning of students. The results confirm the interaction between these variables and their predictive power on performance in mathematics. The study also adds novel considerations related to the function and interaction of cognitive and metacognitive components involved in mathematical performance.

1. Introduction

In the past decades, several scholars in Mathematical Studies (De Corte, 2004; Schoenfeld, 1992, 2005) have shown that mathematical competence depends on factors such as: (1) mathematical knowledge, (2) heuristic methods, (3) meta-knowledge, (4) self-regulatory skills, (5) positive beliefs about oneself in relation to mathematical learning and problem solving, and (6) beliefs about mathematics and mathematical learning.

These authors point out that much of the complexity involved in learning and teaching mathematics is due to the necessary interconnections the student must make between their knowledge and pre-existing skills and attitudes. In this article we will focus on some of the more relevant factors, such as those related to metacognitive self-regulation and control, as well as student beliefs about mathematics and its learning as predictors of mathematical achievement. Metacognitive refers to knowledge regarding the cognitive process itself, as well as the active monitoring and consequent regulation and orchestration of the decisions and processes involved in problem solving. Schoenfeld (1987) described it as follows:

1. Your knowledge about your own thought processes, the description of your own thinking.
2. Self-awareness or self-regulation. How well do you keep track of what you're doing when (for example) you're solving problems, and how well (if at all) do you use the input from such observations to guide your problem solving actions?
3. Beliefs and intuitions. What ideas about mathematics do you bring to your work in mathematics, and how does that shape the way you do mathematics? (p. 190).

This definition illustrates the relevance beliefs and intuition can have. What somebody believes about this discipline determines the way s/he selects a particular direction or method to solving a problem. We agree with Schoenfeld (1985) who perceives mathematical beliefs as “the set of understandings about mathematics that establish the psychological context within which individuals do mathematics and within which work resources, heuristics and control strategies”.

This approach to metacognition adds to its new contents of high functionality, as has been shown in previous studies. The results of our own previous work on these variables make it clear that the scores of subjects in cognitive reflection (i.e., a metacognitive measure) as well as the scores regarding subjects' beliefs about mathematics, and their beliefs regarding their own selves, all correlated positively and significantly with mathematical achievement. It was also found that working memory and reasoning, along with mathematical beliefs, were variables involved in this performance (Gómez-Chacón, García-Madruga, Vila, Elosúa, & Rodríguez, in press).

Another reason to consider these variables is the distinction between two types of cognitive processes in reasoning and judgement: those that run quickly and without conscious deliberation, and the

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slower and more reflective kind. A good part of research in mathematics education has been aimed (explicitly or implicitly) at the relationship between these modes of intuitive and analytical thinking (e.g., Fischbein, 1987; Stavy & Tirosh, 2000). Various studies have explained the conceptual errors in mathematics as consisting of a gap between student intuition and the requirements of the formal system of mathematics. Additionally, the distinction between modes of intuitive and analytical thinking has been comprehensively treated in the theory of dual processes in cognitive psychology (e.g., Evans, 2007; Evans & Over, 1996; García-Madruga, Gutiérrez, Carriedo, Vila, & Luzón, 2007). Authors such as Stanovich and West (2000) refer to these dual processes as “System 1” and “System 2” respectively.

System 1 has been characterized as unconscious, associative, fast, and not linked to individual resources of working memory and fluid intelligence. This system, which humans share to a large extent with other animals, allows individuals to quickly access answers that are often valid, but also lead them to commit mistakes. System 2 is considered conscious, slow, controlled and linked to the individual resources of working memory and fluid intelligence. The performance of System 2 involves the overriding of System 1 and depends on intellectual capability, as well as the disposition and mental styles of individuals.

The role of this dual process theory of reasoning applied to mathematics education is remarkable, and this is a growing field of study (see e.g. Inglis, Mejia-Ramos, & Simpson, 2007; Vamvakoussia, Van Doorenb, & Verschaffel, 2012). There are several themes that have been productive: a) comparative studies with different groups (mathematicians, the general population, college students) used to analyse the influence the so-called belief bias has on responses in conditional inference tasks; b) studies on mathematical processes — such as demonstration — that attempt to identify different types of informal arguments, and the possible differences between expert and novice; and c) studies establishing the intuitive nature of fallacious reasoning in the domain of rational numbers, as well as problems of proportion and persistence in adulthood.

In this paper, we seek to deepen our knowledge about the relationship these belief systems might have when applied to mathematics education, a topic scarcely addressed in the study of dual process theories of reasoning. We would like to identify and understand the positive as well as the negative influences of beliefs that can serve to foster as well as present barriers during cognitive reflection (a metacognitive measure) and reasoning.

The proposal of Evans, like that of Stanovich and West, provides a convergent line of research worth exploring. Evans (see Evans, 2009) proposes a third metacognitive system, System 3, responsible for the activation of working memory, as well as resolving possible conflicts occurring between Systems 1 and 2. Similarly, Stanovich and West (2000); (e.g., Stanovich et al., 2011) propose a tripartite structure. In addition to System 1 and System 2, the so-called reflexive mind exists, and this is responsible for the overall control of the individual's behaviour depending on one's general purpose and goals.

In this article we reflect on a few key variables in the interaction between System 1 (S1) and System 2 (S2) dual processes in mathematics: 1) at the metacognitive level, beliefs provide immediate psychological context and directly affect the performance of subjects in mathematical tasks; 2) cognitive reflection is a measure of metacognitive executive control (regulation) that the subject applies to the resolution of tasks and allows for the inhibition of S1 automatic responses; 3) at the cognitive level, working memory is a fundamental cognitive structure that makes reference to processing and information storage limitations while performing cognitive tasks; and, finally, 3) at the performance level, reasoning abilities are made up of three basic components: deductive inferences (propositional and syllogistic), meta-deductive knowledge, and probabilistic reasoning. Likewise, mathematical achievement scores are at this performance level. Before describing the study and presenting the results, we will review a few theoretical considerations from a historical point of view underlying the variables used in the study.

2. Theoretical aspects

2.1. Belief systems regarding mathematics and learning

Many recent studies have been completed on the essential role of beliefs in learning and teaching mathematics (e.g., Leder, Pehkonen, & Törner, 2002; Maass & Schloeglmann, 2009; Roesken & Casper, 2011). As a unifying framework for the study of belief systems, we refer to the proposal of Op't Eynde, De Corte, and Verschaffel (2002). This proposal allows for a better understanding of the interactions between different types of beliefs, such as reflected in the Mathematics-Related Beliefs Questionnaire (MRBQ) of Op'Eyde and De Corte (2003). These authors refer to the following building blocks for the analysis of the nature and structure of belief systems: the social context, the self, and the object. In previous studies (Gómez-Chacón, Op't Eynde, & De Corte, 2006a, 2006b), we found the need to operationalize the MRBQ questionnaire for the Spanish population. Also, in another project, researchers who designed the CreeMat questionnaire used in the current study also sought to understand the relationship between working memory, reading comprehension, cognitive thinking, deductive reasoning and mathematical belief systems (Gómez-Chacón, García-Madruga, Rodríguez, Vila, & Elosúa, 2011).

Our previous work has led to prioritizing four dimensions in the development of our questionnaire of beliefs: 1) student beliefs about mathematics (MathBe), 2) beliefs about learning and solving math problems (ProsolvBe), 3) student beliefs about themselves (beliefs about the meaning of personal competence in mathematics, that is, the confidence and perception of a student's own ability) (ConfBe), and 4) an affective and behavioural dimension regarding a student's *engagement* in individual mathematical learning (EngBehav). The first three have been discussed in the studies mentioned above, while the fourth dimension does not appear to be integrated in questionnaires such as the MRBQ. In relation to this latter dimension, we would like to point out that we have considered two aspects of engagement in mathematical learning: affective and behavioural engagement. In this sense, experts such as Fredricks, Blumenfeld, and Paris (2004) provide a more comprehensive understanding regarding engagement at school. In our own context regarding learning the discipline of mathematics, we refer only to the school's engagement in the cognitive domain of mathematics. It is within this area that we decided to examine how students feel about the discipline itself (i.e., the affective dimension of commitment) and how they behave when learning the subject (i.e., the engagement expressed in their conduct).

2.2. Cognitive reflection

The proper resolution of arithmetic problems often requires a deep understanding of the problem. The cognitive reflection test (CRT) used in this study is adapted from the evidence presented by Frederick (2005) which, in addition to the three problems used in the initial test, also includes two additional issues proposed and used by the author. The test attempts to evaluate the depth of reasoning of a participant through a simple mathematical reasoning task, similar to the following (problem 1):

A bat and a ball cost \$1.10. The bat costs \$1.00 more than the ball. How much does the ball cost? ___ cents.

Faced with this kind of mathematical problem, subjects tend to give an impulsive response that comes readily to mind: “10 cents”. However, this answer is wrong, as a little reflection will make clear: the difference between \$1.00 and 10 cents is 90 cents and not \$1.00, as the problem