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Solution strategies and adaptivity in multidigit division in a choice/no-choice experiment: Student and instructional factors

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A B S T R A C T

Adaptive expertise in choosing when to apply which solution strategy is a central element of current day mathematics, but may not be attainable for all students in all mathematics domains. In the domain of multidigit division, the adaptivity of choices between mental and written strategies appears to be problematic. These solution strategies were investigated with a sample of 162 sixth graders in a choice/no-choice experiment. Children chose freely when to apply which strategy in the choice condition, but not in the no-choice conditions for mental and written calculation, so strategy performance could be assessed unbiasedly. Mental strategies were found to be less accurate but faster than written ones, and lower ability students made counter-adaptive choices between the two strategies. No teacher effects on strategy use were found. Implications for research on individual differences in adaptivity and the feasibility of adaptive expertise for lower ability students are discussed.

1. Introduction

Learning and problem solving are characterized by the use of a variety of strategies at every developmental stage (Siegler, 2007). Children's and adults' strategy use has been investigated for cognitive tasks concerning diverse topics such as class inclusion (Siegler & Svetina, 2006), analogical reasoning (Tunteler, Pronk, & Resing, 2008), and digital gaming (Ott & Pozzi, 2012). A well-studied area of investigation in solution strategy research is strategy use for arithmetic problems. Many studies have been conducted on strategies in elementary addition, subtraction, multiplication and division (e.g., Barrouillet & Lépine, 2005; Imbo & Vandierendonck, 2007; Mulligan & Mitchelmore, 1997; Van der Ven, Boom, Kroesbergen, & Leseman, 2012), which concern operations in the number domain up to 100 that are taught in the lower grades of primary school. However, there is a notable scarcity of research on strategy use of higher grade students on more complex arithmetic problems (though not an absence; see for example Van Putten, Van den Brom-Snijders, & Beishuizen, 2005; Selter, 2001; Torbeyns, Ghesquière, & Verschaffel, 2009b). This more advanced arithmetic is called multidigit arithmetic, as it involves larger numbers and decimal numbers. Multidigit arithmetic is particularly interesting with regard to strategy use, as the higher complexity of the problems allows for the use of a wider range of strategies.

1.1. Solution strategies and adaptivity

To chart strategy use for a given domain, Lemaire and Siegler (1995) proposed a general framework consisting of four aspects of strategic competence: strategy repertoire (which strategies are used); frequency (how often each strategy in that repertoire is chosen for use); efficiency (performance with use of each strategy); and adaptivity (the appropriateness of a choice for a strategy given its relative performance). While the first three aspects of the framework are quite straightforward, the aspect of adaptivity has been conceptualized in various ways by different researchers. Verschaffel, Luwel, Torbeyns, and Van Dooren (2009) reviewed the existing literature on this topic and identified three factors that play central roles in the different conceptualizations.

First there is the role of task variables, which concern the adaptation of strategy choices to problem characteristics. For example, for a problem such as 62−29 the adaptive strategy choice could be defined as compensation (Blöte, Van der Burg, & Klein, 2001): the problem can be greatly simplified by rounding the subtrahend 29 to 30, and then compensating for this after the subtraction (62−30+1). Second there is the role of subject

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variables, which concern the adaptation of strategy choices to strategies' relative performance for a particular individual (for a particular problem), such as in the Adaptive Strategy Choice Model (ASCM; Siegler & Shipley, 1995). Third there is the role of context variables, which can be both in the direct context of the task (such as time restrictions) and in the broader sociocultural context (such as the value placed on accuracy versus speed). Verschaffel et al. (2009) combine all three factors (calling for more research attention for context variables especially) in defining a strategy choice as adaptive when it is most appropriate for a particular problem for a particular individual, in a particular sociocultural context.

A second issue in determining adaptivity is that often there is not one unequivocal best performing strategy, as the most accurate strategy is not always also the fastest. This can be addressed by combining speed and accuracy in a definition of the best performing strategy as the one that leads to the correct solution the fastest (e.g., Luwel, Onghena, Torbeyns, Schillemans, & Verschaffel, 2009; Torbeyns, De Smedt, Ghesquière, & Verschaffel, 2009a; Kerkman & Siegler, 1997). Yet, even with this definition, researchers tend to consider accuracy and speed separately in their statistical analyses in practice (with the exception of Torbeyns, Verschaffel, & Ghesquière, 2005).

1.2. Adaptive expertise in mathematics education

Debates of its exact definition aside, adaptivity has become more and more important in the educational practice of primary school mathematics. Reforms in mathematics education have taken place in various countries over the past decades (Kilpatrick, Swafford, & Findell, 2001) and they have reshaped the didactics for multidigit arithmetic from prescribing a fixed algorithmic strategy per problem type to building on students' own strategic explorations (Gravemeijer, 1997). For students, this means that performing well now requires more than perfecting the execution of a limited set of algorithmic strategies, because choosing the best performing strategy for solving a problem is also necessary. Adaptive expertise has become a central element of education: students should have an array of strategies at their disposal, that they can use efficiently, flexibly and creatively when they solve problems (Verschaffel et al., 2009). Investigations differ in their findings of whether such adaptivity is attainable for everyone: some have found evidence of a general adaptivity of strategy choices (e.g., Siegler & Lemaire, 1997; Torbeyns et al., 2005), while others found it only for students with a high mathematical ability (e.g., Hickendorff, Van Putten, Verhelst, & Heiser, 2010; Torbeyns, Verschaffel, & Ghesquière, 2006), and some not at all (e.g., Torbeyns, De Smedt, et al., 2009a).

In addition to providing more space for informal strategies, the reforms introduced new standardized approaches for the more complex multidigit problems. With traditional algorithms the large numbers in such problems are considered one or two digits at a time, without an appreciation of the magnitude of those digits in the whole number being necessary, while new approaches place more focus on the whole number (as such, the former approaches have been labeled 'digit-based' and the latter 'whole-number-based'; Van den Heuvel-Panhuizen, Robitzsch, Treffers, & Köller, 2009). Especially for multidigit division, digit-based algorithms (e.g., long division) have been de-emphasized or even abandoned in favor of whole-number-based approaches (e.g., partial quotients; Buijs, 2008; Scheltens, Hemker, & Vermeulen, 2013). Table 1 provides examples of digit-based and whole-number-based approaches for division: while they both consist of standardized steps with a schematic notation, the digit-based algorithm breaks the dividend up into digits (e.g., in Table 1, the 85 part of 850 is considered separately when subtracting 75, and the rest of the dividend is only considered in a later step), whereas the whole-number-based algorithm considers the dividend as a whole (e.g., 250 is subtracted from 850).

However, dismissing a digit-based algorithm does not necessarily mean that a whole-number-based algorithm will be used instead; an increase in the use of more informal, non-algorithmic strategies is also possible, even though they may be less suited for challenging problems. For example, the decrease in the use of the digit-based division algorithm in Dutch national assessments from 1997 to 2004 was paired by an almost equal increase in answering problems without writing down any calculations (Van Putten, 2005), which should be interpreted as mental calculation (Hickendorff et al., 2010). This switch from written to mental calculation turned out to be very unfortunate, as the probability for a student to solve a division problem accurately was drastically lower with mental than with written calculation (Hickendorff, Heiser, van Putten, & Verhelst, 2009), and the overall performance level on multidigit division decreased sharply from 1997 to 2004 (Janssen, Van der Schoot, & Hemker, 2005). This trend over time of an increasing percentage of students choosing an inaccurate strategy suggests that the reform goal of adaptive expertise may not be feasible for some domains of mathematics.

1.3. The present study

The present study therefore constitutes an in-depth experimental investigation of adaptivity in this domain of mathematics that was particularly affected by the reforms: multidigit division. An experimental approach is necessary, because performance estimates of strategies may be biased by so-called selection effects (Siegler & Lemaire, 1997): for example, though mental strategies produce a low percentage of correct solutions for multidigit division problems, this performance estimate may be biased because of the mathematical ability level of the students who choose to use this strategy or because of the difficulty of the problems it is applied to. If mental calculation were used equally by all types of students on all types of problems, then a different estimation of its performance could very well result. Hickendorff et al. (2010) experimentally compared a condition in which students freely chose when to write down calculations and one in which they had to write down calculations for every problem, and found that written calculation was at least as accurate or more accurate than mental calculation, especially for weak students. Mental calculation, however, was only observed in this study when spontaneously chosen and therefore performance estimates were biased by selection effects. In addition, only accuracy and not solution times were measured, so the role of speed in strategy choices and adaptivity remained unclear.

The present study addresses these two issues by experimentally investigating students' spontaneous strategy choices for multidigit division and both their accuracy and speed with required written and required mental calculation. The participants are sixth graders, because the radical changes in performance and strategy use were demonstrated for this age group in the aforementioned large-scale assessments. The aim of the present study is to systematically chart the four aspects of strategic competence of Lemaire and Siegler (1995) – repertoire, frequency, efficiency and adaptivity – with special attention to adaptivity, because of its high relevance to mathematics education and to multidigit division specifically. This was done using the choice/no-choice paradigm introduced by Siegler and Lemaire (1997) to allow for the unbiased assessment of strategy performance characteristics, that has since been applied in numerous solution strategy investigations (e.g., Imbo & Vandierendonck, 2007; Lemaire & Lecacheur, 2002; Torbeyns et al., 2005).

This design consists first of a choice phase in which participants freely choose between strategies in solving a set of problems. This