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The preparatory effects of problem solving versus problem posing on learning from instruction

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ABSTRACT

Two randomized-controlled studies compare the preparatory effects of problem-solving versus problem-posing on learning from subsequent instruction. Students engaged in either problem-solving (where they generated solutions to a novel problem) or problem-posing (where they generated problems, and where possible, the associated solutions) prior to learning a novel math concept. Study 1 found that problem-posing prior to instruction resulted in significantly better transfer to novel problems than problem-solving, without any significant difference in procedural knowledge and conceptual understanding. Study 2 further showed that when problem-posing was designed to focus only on the generation of problems without the solutions, problem-solving prior to instruction resulted in better conceptual understanding than problem-posing. However, the transfer effect remained in favor of problem-posing, albeit weaker than in Study 1. These findings suggest that although solution generation prior to instruction plays a critical role in the development of conceptual understanding and transfer, generating problems can further enhance transfer.

1. Introduction

There is now a growing body of evidence that preparatory activities such as generating solutions to novel problems *prior* to instruction can help students learn better from the instruction (Kapur & Rummel, 2012). Evidence comes not only from quasi-experimental studies conducted in the real ecologies of classrooms (e.g., Kapur, 2012, 2013; Schwartz & Bransford, 1998; Schwartz & Martin, 2004), but also from controlled experimental studies (e.g., DeCaro & Rittle-Johnson, 2012; Kapur, 2014; Loibl & Rummel, 2013, 2014; Roll, Aleven, & Koedinger, 2011; Schmidt & Bjork, 1992; Schwartz, Chase, Oppezzo, & Chin, 2011). For example, in a study with eight-grade students, Schwartz et al. (2011) compared students who invented solutions with contrasting cases before receiving instruction on the concept of density with those who were instructed first and then practiced with the same cases. They found that the guided invention activities prepared students to learn the deep structure of density better than those who received instruction first. Likewise, DeCaro and Rittle-Johnson (2012) had second-to fourth-grade students solve unfamiliar math problems on number sentences before or after receiving instruction on number sentences. Once again, students who solved problems first developed better conceptual understanding than those who first received instruction. More recently, in a randomized-controlled experiment with ninth-graders learning the concept of Standard Deviation (SD), Kapur (2014) had students individually generate solutions to a novel problem before or after receiving instruction. He found that students who engaged in problem-solving prior to instruction demonstrated significantly better performance on conceptual understanding and transfer than those who engaged in problem-solving after instruction.

There are several interdependent mechanisms underpinning the preparatory effects of problem-solving prior to instruction. First, starting with problem-solving may be better at activating and differentiating relevant prior knowledge provided students are able to use their priors to generate sub-optimal or even incorrect solutions to the problem (DeCaro & Rittle-Johnson, 2012; Schwartz et al., 2011). Because students can rely only on their prior knowledge to generate solutions, the nature of these solutions provides a measure of the types of knowledge that was activated, and how this knowledge is relevant in relation to the targeted concept (Kapur, 2014; Loibl & Rummel, 2014; Wiedmann, Leach, Rummel, & Wiley, 2012). Second, prior knowledge activation may in turn afford more opportunities for students to: a) notice the inconsistencies in and realize the limits of their prior knowledge

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(DeCaro & Rittle-Johnson, 2012; Loibl & Rummel, 2014), and b) compare and contrast student-generated solutions and correct solutions during subsequent instruction, thereby helping students' to attend to and better encode critical features of the new concept (Kapur, 2014; Schwartz et al., 2011). Finally, besides the cognitive benefits, problem-solving prior to instruction may also have affective benefits of greater learner agency, as well as engagement and motivation to learn the targeted concept (Belenky & Nokes-Malach, 2012; Bielaczyc & Kapur, 2010; Kapur & Bielaczyc, 2012; diSessa, Hammer, Sherin, & Kolpakowski, 1991).

Taken together, these mechanisms collectively point to the efficacy of preparatory activities that engage students in solving novel problems prior to instruction. That said, these preparatory activities present students with contexts where the problem is given to the students. An equally, if not more, important mathematical skill is to generate problems in the first place (Jay & Perkins, 1997; Silver, 1994; also see special issue on problem-posing in Educational Studies in Mathematics, 2013). The proposition being: students need to be provided opportunities for both problem-solving and problem-posing. In math education literature, problem-posing has been conceptualized as either the reformulation of a given problem, or an extension of a given problem, or the generation of new problems from a given situation (Silver, 1994). In the studies reported in this paper, problem posing refers to the generation of new problems from a given situation.

Notwithstanding a strong emphasis on problem-posing as an instructional and learning goal, empirical evidence in math education research remains largely descriptive in nature. Consequently, the efficacy of engaging in problem-posing to learn new math concepts has not been systematically tested.

Therein lies the purpose of this paper: to examine the relative efficacy of problem-solving versus problem-posing as a preparatory activity for learning from subsequent instruction. I start with a review of research on problem-posing in math learning, followed by articulating hypotheses comparing the preparatory mechanisms of problem-posing with those of problem-solving. I then report findings from two randomized-controlled experiments, and end by discussing the findings and drawing implications for math learning and instruction.

2. Problem-posing in math learning

A review of research in math learning revealed that past research is largely theoretical or descriptive in nature (e.g., English, 1998; Silver & Cai, 1996). Purported benefits of problem-posing include the development of greater learner agency and reflection (Kilpatrick, 1987), responsibility and insight that helps understanding and reduces anxiety (Brown & Walter, 2005), and ownership and engagement that can potentially help math learning (Cunningham, 2004; Perrin, 2007; Silver, 1994). Because evidence for these benefits comes mainly from descriptive studies, the absence of any comparison or controls limits what one can infer from these studies about the preparatory effects of problem-posing.

The closest experimental comparison in math learning, though not from a preparatory lens, comes from the work of Sweller and colleagues (Mawer & Sweller, 1982; Sweller, Mawer, & Howe, 1982; Sweller, Mawer, & Ward, 1983). They showed that a reduction in or elimination of goal specificity of a given problem situation, a move that essentially requires students to pose and answer as many questions for solving a particular problem, helped schema acquisition for problem-solving. For example, in geometry (Sweller et al., 1983), students were first instructed on the targeted geometry concepts before being assigned to solve either no-goal or goal-specific problems. The no-goal problem required students to ask and answer as many unknown angles in a geometry diagram. The goal-specific problem required students to solve for a particular unknown in the same diagram. As hypothesized, findings suggested that the no-goal students performed better than goal-specific students on subsequent problem-solving on similar geometry problems. Note however that students in these studies were first taught the concept before they solved problems, with or without goals. Hence, whereas these studies may well speak to the cognitive load mechanisms of using problem-posing as a problem-solving strategy *after* learning a concept, they do not directly speak to the preparatory mechanisms of problem-posing *before* learning and for learning a new concept.

In a more recent quasi-experimental study with in-service elementary mathematics teachers, Toluk-Ucar (2009) reported that the treatment group who experienced posing and discussing fraction problems during a 6-week intervention performed significantly better on the posttest on understanding of fractions than the comparison group who followed the traditional approach of peer teaching lessons on fractions that they had designed. Further analysis revealed that the treatment group generated problems that were mathematically more appropriate, modeled greater variance in the meaning of operations, and gave conceptually stronger explanations for pictorial representations in their problems.

Although the studies of Sweller and colleagues as well as Toluk-Ucar suggest an efficacy of problem-posing for learning math, participants in these studies either already had substantial knowledge of the targeted concepts or had formally been taught those concepts before they engaged in problem-posing. If participants already have knowledge of the targeted concepts, it becomes hard to infer anything about the efficacy of problem-posing as a preparatory activity for learning new concepts. From the lens of preparatory activities (Schwartz & Martin, 2004; Schwartz et al., 2011), one would want to examine how problem-posing can prepare students to learn new concepts in the first place, where participants engage in problem-posing *before* learning the targeted concepts.

3. Preparatory mechanisms of problem-solving versus problem-posing

I start by comparing the preparatory mechanisms of problem-posing with the four preparatory mechanisms of problem-solving articulated earlier. I then put forth two additional mechanisms relevant to the comparison.

First, one could expect problem-posing to afford greater prior knowledge activation and differentiation than problem-solving because problem-posing does not constrain the problem-space as problem-solving does. By greater, as defined in previous work (e.g., Kapur, 2013, 2014), I mean the numbers of problems and solutions students are able to generate. Consequently, the solution space within which problem-posers can activate prior knowledge in search for potential problems and solutions is greater (Frensch & Funke, 1995).

However, there is likely to be a trade-off between greater knowledge activation and how relevant the activated knowledge is. By relevance, I mean whether the activated knowledge is conceptually related to the targeted concept. Therefore, the extent to which students benefit from greater prior knowledge activation may be contingent upon whether such activation is relevant to the learning of the targeted concept. Research on the role of goal specificity and learning suggests that the benefits of problem-posing (akin to having no goal or low goal specificity) are derived mainly if the lack of a goal actually affords students the opportunities to attend to the deep structure of the problem and solution spaces (Burns & Vollmeyer, 2002; Miller, Lehman, & Koedinger,