Learning and Instruction 34 (2014) 74–85

Contents lists available at ScienceDirect

Learning and Instruction

journal homepage: www.elsevier.com/locate/learninstruc

Knowing what you don't know makes failure productive☆

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ARTICLE INFO

Article history:
Received 6 June 2013
Received in revised form
18 August 2014
Accepted 26 August 2014
Available online 16 September 2014

Keywords:
Comparison
Delaying instruction
Knowledge gap
Learning from errors
Productive failure

ABSTRACT

To progress from intuitive ideas to deep conceptual understanding, students need to become aware of gaps in their ideas. Attempting to solve problems prior to instruction may lead to a global awareness of knowledge gaps (i.e., awareness without being able to identify which specific component is lacking). These gaps may subsequently be specified by comparing students' solutions to the canonical solution. In our first experiment, the teacher highlighted specific gaps by comparing typical student solutions to the canonical solution before or after problem solving. The second experiment varied the factors *form of instruction* (with or without student solutions) and *timing of instruction* (problem-solving prior to or after instruction). Problem-solving prior to instruction triggered a global awareness of knowledge gaps. While this was beneficial for learning when combined with instruction with student solutions, our results indicate that comparing student solutions during instruction to specify the gaps is the most relevant factor.

1. Introduction

A major goal in education is to support learners in the transition from their intuitive, often erroneous or incomplete ideas to a deep understanding of a concept. This deep understanding of the underlying principles and the interrelation of the knowledge components is called conceptual knowledge (Rittle-Johnson, Siegler, & Alibali, 2001). Conceptual knowledge is reflected, for example, in principle-based reasoning or in the ability to connect different representations. In contrast to procedural skills, which can be learned by solving practice problems after learning the canonical procedure (Rittle-Johnson et al., 2001; Sleeman, Kelly, Martinak, Ward, & Moore, 1989), conceptual knowledge can be acquired by elaboration and sense-making processes (e.g., Koedinger, Corbett, & Perfetti, 2012). These sense-making processes enable learners to relate new information to prior knowledge and intuitive ideas. Approaches that elicit prior knowledge and intuitive ideas, and that make limitations of the existing knowledge structure as well as the connection to new information explicit, may therefore be especially valuable for facilitating conceptual knowledge.

1.1. Problem-solving prior to instruction

One approach to elicit prior knowledge and intuitive ideas is to ask students to explore (mathematical) problems *prior* to instruction (Schoenfeld, 1992). Indeed, recent studies have shown potential benefits of problem-solving prior to instruction for the acquisition of conceptual knowledge (Kapur, 2010, 2012; Kapur & Bielaczyc, 2012; Loibl & Rummel, 2014; Roll, Aleven, & Koedinger, 2009, 2011; Schwartz & Martin, 2004; Westermann & Rummel, 2012). In these studies, students who first solved problems regarding a hitherto unknown concept before receiving instruction outperformed those who received direct instruction (i.e., instruction without previous problem solving) in terms of conceptual knowledge, without compromising the acquisition of procedural skills. In general, it has been argued that problem-solving prior to instruction prompts students to activate their prior knowledge and intuitive ideas about the domain in question (e.g., Kapur & Bielaczyc, 2012; Schoenfeld, 1992). However, the implementation of the initial problem-solving phase and the subsequent instruction phase has differed across studies. In the so-called *Invention* approach (e.g., Roll et al., 2009, 2011; Roll, Holmes, Day, & Bonn, 2012; Schwartz & Martin, 2004), problems are presented in the form of contrasting cases, which are series of small datasets (e.g., for a task on variance: Player A scored 10 10 10, Player B scored 8 10

☆ Following the guidelines of the Publication Manual of the American Psychological Association, we would like to inform the reader that a paper focusing on a different research question (The impact of guidance during problem-solving prior to instruction on students' inventions and learning outcomes) which has been published in Instructional Science, includes analyses on parts of the dataset also used in this paper.

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http://dx.doi.org/10.1016/j.learninstruc.2014.08.004
0959-4752/

12). Every pair of datasets differs regarding one feature at a time, while all other features are held constant. In the above example, the mean of the goals is the same for both players, but the range differs. Thereby, students are likely to notice that the deviation of the goals matters and devise a solution that takes this specific feature into account. Other features are made salient by comparing the next pair of datasets. By asking students to generate a solution that works for all sets of contrasting cases, they are prompted to take all features into account. Applying their solution to the contrasting cases provides feedback regarding specific gaps in the solution. This process supports them in generating a solution that goes beyond their initial intuitive ideas. In contrast, in the so-called *Productive Failure* approach (e.g., Kapur, 2010, 2012), problems are presented in the form of rich datasets and students are asked to devise different solutions to elicit a broad range of prior knowledge and intuitive ideas. The rich datasets do not make salient the features that are relevant for finding the solution. Students are therefore not enabled to guess elements of the canonical solution and are thus unlikely to generate solutions beyond their intuitive ideas. As students fail to invent a canonical solution (e.g., Kapur & Bielaczyc, 2012), the struggle with the problem at hand most likely triggers a global awareness of knowledge gaps (i.e., an awareness that they have knowledge gaps without being able to specify which component they are lacking).

In both scenarios, students usually fail to devise a canonical solution during the problem-solving phase. Therefore, an instructional phase follows. In the Invention approach, the instruction focuses on the canonical solution. The assumption is that students are likely to realize that the canonical solution accounts for all relevant features, as it works for all contrasting cases. In Productive Failure, the instruction explicitly builds on typical student solutions, illustrating their limitations and gaps in a first step. As the features relevant for the solution of the problem were not made salient, this phase may be necessary to create awareness of the specific gaps in students' intuitive ideas. Only afterwards does the teacher introduce the canonical solution and explain how the canonical solution resolves the identified gaps. As the different ways of implementing both phases have not yet been compared empirically, the processes described above can only be hypothesized on a theoretical basis.

We argue that by raising students' awareness of their knowledge gaps, both types of problem-solving prior to instruction approaches support students in transitioning from their first intuitive ideas to deep conceptual knowledge. Through two empirical studies, we aim to shed light on this assumption by investigating two factors that may lead to such an awareness of knowledge gaps: first, problem-solving before as opposed to after instruction, and second, instruction that builds on typical student solutions to make the gaps salient as opposed to instruction focusing only on the canonical solution. Before describing our studies in more detail, we present relevant literature on processes triggered by the awareness of knowledge gaps, and we discuss how these processes may be facilitated in problem-solving prior to instruction settings. We focus in particular on the Productive Failure approach, as it allows a distinction to be made between a global awareness of knowledge gaps (elicited during problem solving) and an awareness of specific knowledge gaps (elicited during instruction).

1.2. Awareness of knowledge gaps

Research has shown that students process a canonical solution more deeply when they are aware of their own impasses and errors (VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003). This holds true not only for errors that students make themselves (cf. VanLehn et al., 2003), but also for possible errors that are likely to occur based on typical knowledge gaps. For instance, learning can be fostered by warning students of possible errors when working on a specific task before presenting an instructional explanation (Acuña, García-Rodicio, & Sánchez, 2010; Sánchez, García-Rodicio, & Acuña, 2009) or explicitly addressing typical errors and preconceptions in an instructional text (e.g., Diakidoy, Kendeou, & Ioannides, 2003). According to these researchers, the warning helps students to become aware of their own knowledge gaps. In consequence, students perceive subsequent instructional explanations as relevant to repair these gaps and are thus more likely to process the instruction deeply.

This notion can also be found in several theoretical (and empirically tested) models on problem solving: VanLehn (1999) describes the *impasse-repair-reflect process* as part of the Cascade learning model. If the cognitive system (i.e., the learner) reaches an impasse, it applies strategies to repair the impasse. Subsequent instructional explanations can help students to repair this impasse if the explanations meet the students' detected impasse. A similar process is described in Chi's (2000) work on repairing mental models: In her *imperfect mental model view*, she states that learners' first mental models usually differ from normative models. In order to repair these non-canonical mental models, students first have to detect flaws in their models before they can actively engage in processes which repair their models. In the context of problem-solving prior to instruction, students' first mental models are represented in their intuitive solution attempts. As these solution attempts are usually erroneous or incomplete (cf. Kapur, 2012; Kapur & Bielaczyc, 2012), students may become aware of their knowledge gaps in a general way. Comparing the solution attempts and contrasting them to the canonical solution during instruction helps them to detect differences in a more specific manner (e.g., Smith, diSessa, & Roschelle, 1994). In other words, their knowledge gaps are specified. Identifying very specific knowledge gaps should ease the process of repairing flawed mental models. The assumption that students are motivated to resolve their gaps (Chi, 2000; VanLehn, 1999) is supported by Belenky and Nokes-Malach (2012), who showed that students adopted a mastery orientation after failing to generate a canonical solution in an Invention setting (in other words, they strove to learn the canonical solution) and acquired a better understanding during subsequent instruction (cf. also Belenky & Nokes, 2009; Nokes & Belenky, 2011).

In order to resolve the detected knowledge gaps, students need to focus on the relevant features of the new learning content in order to process these components deeply (Renkl & Atkinson, 2007), thereby resolving their gaps. The relevant features can be highlighted by means of providing students with contrasting cases (as described with regard to the Invention approach) or by helping students to detect differences between prior ideas and the canonical solution through a comparison of erroneous and correct worked examples (e.g., Durkin & Rittle-Johnson, 2012; Große & Renkl, 2007). This assumption is further supported by Gadgil, Nokes-Malach, and Chi (2012): In their study, learners who compared their flawed models to expert models were more likely to repair their incorrect model than learners who only self-explained expert models.

1.3. Limitations of classic productive failure studies

How do these assumptions explain the findings of the classic Productive Failure studies (e.g., Kapur, 2010, 2012)? As argued above, the initial problem-solving phase may prompt a global awareness of knowledge gaps, as students do not yet know the canonical solution. However, it remains unclear whether this global awareness of knowledge gaps is sufficient to trigger beneficial processes to repair the gaps. Subsequent instruction that builds on