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Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents

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ABSTRACT

Numerical understanding and arithmetic skills are easier to acquire for whole numbers than fractions. The *integrated theory of numerical development* posits that, in addition to these differences, whole numbers and fractions also have important commonalities. In both, students need to learn how to interpret number symbols in terms of the magnitudes to which they refer, and this magnitude understanding is central to general mathematical competence. We investigated relations among fraction magnitude understanding, arithmetic and general mathematical abilities in countries differing in educational practices: U.S., China and Belgium. Despite country-specific differences in absolute level of fraction knowledge, 6th and 8th graders' fraction magnitude understanding was positively related to their general mathematical achievement in all countries, and this relation remained significant after controlling for fraction arithmetic knowledge in almost all combinations of country and age group. These findings suggest that instructional interventions should target learners' interpretation of fractions as magnitudes, e.g., by practicing translating fractions into positions on number lines.

1. Introduction

Understanding fractions is crucial for mathematics learning: It not only requires a deeper understanding of numbers than is ordinarily gained through experience with whole numbers, it is also predictive for students' mathematical achievement years later (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; Siegler et al., 2012). Despite increasing research interest in the domain of fractions, almost all studies of the role of fraction magnitude understanding in mathematics learning have been conducted in the U.S., limiting the generality of the findings to only U.S. students and adults. This study aims to deepen our understanding of the pivotal role of fraction magnitude understanding for students' general math achievement in three countries on three different continents that differ greatly in cultural and educational practices.

1.1. The integrated theory of numerical development

Our starting point was Siegler, Thompson, and Schneider's (2011) *integrated theory of numerical development*.[1] As discussed there, current theories of numerical development fail to integrate whole numbers and fractions within a single framework (e.g., Geary, 2006; Leslie, Gelman, & Gallistel, 2008; Wynn, 2002). Although these theories differ in many particulars, they all posit a gap between an early developing, "natural" understanding of whole numbers and a later developing, flawed, limited, or hard-won understanding of fractions. To the extent that relations between whole numbers and fractions are posited, the earlier

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[1] It should be noted that knowledge acquisition, instruction, and cognitive development are closely intertwined both in real life and in the integrated theory of numerical development. Therefore, the word *development* in the latter theory's name should be understood in its broadest meaning, i.e., as integrating – rather than excluding – knowledge acquisition and instruction as important influences on people's numerical development. However, for readability reasons, the authors decided not to include all three sources of competence growth in the theory's name and refer to the, in the learning sciences, well-known and frequently used term *development*.

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developing understanding of whole numbers is said to interfere with the later developing understanding of fractions. For instance, according to conceptual change theories (DeWolf & Vosniadou, in this issue; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2010), children form an initial theory of number as counting units before they encounter fractions, and draw heavily on this initial understanding of number to make sense of fractions. Children's faulty generalization of understanding of number as counting units interferes with their learning about fractions, a phenomenon often referred to as the "whole number bias" (Ni & Zhou, 2005).

Siegler, Thompson, and Schneider's (2011) *integrated theory of numerical development* recognizes these important differences between learning of whole numbers and fractions, but also emphasizes a crucial continuity that unites their acquisition – steadily expanding understanding of the connection between numbers and their magnitudes. Within this perspective, development of understanding of rational numbers involves both a gradual expansion of the range of whole numbers whose magnitudes are understood (from smaller to larger) and a conceptual change from an initial understanding of numbers in terms of characteristic features of whole numbers to a later understanding of rational numbers in terms of a single defining feature, their magnitudes (see Wu, 2001, 2009, for a similar argument).

The integrated theory differs from conceptual change theories in two main ways. One is in its recognizing the *positive* role of whole number magnitude knowledge in learning fractions, as indicated by longitudinal relations between first graders' knowledge of whole number magnitudes and 7th and 8th graders' knowledge of fraction magnitudes and fraction arithmetic, even after statistically controlling for the IQ, working memory, socio-economic status, race, and other relevant variables (Bailey, Siegler, & Geary, 2014). The second main difference between the integrated theory and conceptual change theories of fraction knowledge is that the integrated theory views interference from whole number knowledge as only one of several sources of difficulty in learning fractions. Evidence for this view comes from findings that despite whole number errors, such as $1/2 + 2/3 = 3/5$ being common, confusion with other fraction operations, such as the confusion between fraction addition and fraction multiplication evident in $1/3^*2/3 = 2/3$, can be even more common (Siegler & Pyke, 2013).

Within this integrated theory, the reason why fractions are more difficult to learn than whole numbers is the same reason why fractions are crucial to numerical development. A fraction is a ratio or division of two whole numbers, numerator and denominator, and is thus considerably more complex than a single whole number. Whole numbers have unique predecessors and successors, but this is not true of fractions. Multiplying a whole number always leads to a larger number and dividing a whole number always leads to a smaller number, but again this is not true of fractions. Thus, generating a mature understanding of rational numbers requires understanding both the one property that all rational numbers share – that they have magnitudes that can be located and ordered on number lines – and understanding that other properties that unite whole numbers do not unite rational numbers.

Consistent with this theory, Siegler et al. (2011) found strong relations between U.S. 6th and 8th graders' fraction magnitude understanding and their general mathematics achievement, even when their mutual relation to fraction arithmetic was statistically controlled. However, these and other data on this topic were collected almost exclusively in the U.S. It thus remains an open question whether the findings are due to the proposed general cognitive learning mechanisms of the theory or to specific properties of the U.S. cultural and educational system (e.g., cultural beliefs about mathematics, teacher training, time spent on mathematics, mathematics curricula).

1.2. Previous studies on fraction understanding

Although research interest in students' acquisition of fraction knowledge and skill has increased in recent years, such studies are still far less numerous than studies of whole number understanding. However, the limited number of studies of fractions and the much larger number of studies of whole numbers have revealed highly similar relations among magnitude understanding, arithmetic and general mathematics achievement (Siegler, Fazio, Bailey, & Zhou, 2013).

The same types of behavioral methods have proved useful for investigating fraction as whole number magnitudes: magnitude comparison tasks, in which participants compare the magnitudes of two whole numbers or fractions and indicate the larger one, and number line estimation tasks, in which participants indicate the position of a given whole number or fraction on an empty number line with clearly indicated start and end point. Studies using these methods have consistently revealed that, as with whole number magnitude representations, the precision of fraction magnitude representations differs greatly between and within individuals, depending on students' (instructional) experiences with fractions and the size of the fractions (Siegler & Pyke, 2013; Siegler et al., 2011). Also as with whole numbers, fraction magnitude understanding has proved to be quite strongly correlated with other aspects of mathematics learning. On top of this correlational evidence, recent investigations provide evidence for predictive relations between earlier fraction magnitude understanding and subsequent knowledge of fraction arithmetic, algebra and overall math achievement (Bailey et al., 2012; Booth & Newton, 2012; Siegler et al., 2012). To cite one example, Siegler et al. (2012) demonstrated that 5th graders' fraction knowledge predicts their mastery of algebra and overall mathematics achievement in high school, 5 or 6 years later, even after controlling for IQ, reading achievement, working memory, family income and education, and whole number knowledge. The same relations were found in both U.K. and U.S. longitudinal samples. Moreover, Fuchs et al. (2013) demonstrated that instruction focused on fraction magnitude understanding improved not only understanding of fraction magnitudes but also fraction arithmetic proficiency among children with mathematics learning difficulties. Taken together, these results indicate that magnitude representations are as central to knowledge of fractions as to knowledge of whole numbers.

However, to the best of our knowledge, all previous behavioral studies of the role of fraction magnitude understanding in mathematics learning have been conducted in the U.S. – with the one exception of Siegler et al. (2012), which included both U.S. and U.K. samples. This raises questions about the generality of the findings and (consequently) the applicability of the integrated theory of numerical development to populations in other countries and continents. Differences in instructional methods, curricular devices, teacher expertise, and students' absolute levels of achievement might all limit the generality of the findings that have been viewed as supporting the integrated theory. Therefore, in the present study, we investigated students' fraction understanding in three countries with quite different instructional methods and teaching practices: Belgium (Flanders), China, and the U.S.

1.3. Differences in teacher knowledge and instructional practices in mathematics

International investigations of (prospective) teachers' knowledge, instructional practices and student performances in the