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The representation of fraction magnitudes and the whole number bias reconsidered

Melissa DeWolf [a], Stella Vosniadou [b, *]

[a] Department of Psychology, University of California, Los Angeles, 405 Hilgard Ave., Los Angeles, CA, 90095-1563, USA
[b] Department of Philosophy and History of Science, National and Kapodistrian University of Athens, Athens, Greece

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ABSTRACT

Two experiments investigated the whole number bias in the representation of fraction magnitudes with adults. A fraction magnitude comparison task was used where half of the comparisons were consistent with whole number ordering and the other half were not. Distance effects were found in Experiment 1 indicating that participants were comparing the magnitude of the whole fraction rather than just the parts. However, accuracy and response time also depended on the comparisons' consistency with whole number ordering. Experiment 2 manipulated the distance between the fraction pairs and showed that the whole number effect was strongest when the distance between the fraction pairs was very small. The results suggest that even skilled adults do not always have direct access to a fraction's magnitude on the number line. When the magnitudes are especially close together, adults may rely on alternative implicit or explicit strategies, such as examining the whole number parts, to evaluate the comparison.

1. Introduction

1.1. Mental representation of fraction magnitudes

The purpose of the research presented in this article is to investigate the nature of the assumed representation of fractions in the fully developed number system of skilled adults. More specifically, the question has been raised as to whether adults can process fractions as a whole through direct access to their magnitude, or rely on the magnitude of their component parts (numerator and denominator). The assumption is that if adults can process fractions as a whole they have a representation of fractions based on the continuum of real numbers, allowing direct access to their value. On the other hand, if the processing of fractions is componential, this would imply that the representation of fractions in adults is based on a system of discrete, integer numbers that are used to compute the integrated magnitude (Bonato, Fabbri, Umiltà, & Zorzi, 2007; Kallai & Tzelgov, 2012).

In order to assess the extent to which adults represent whole number magnitudes on a mental number line, researchers have largely relied on magnitude comparison tasks (Dehaene, Dupoux, & Mehler, 1990; Holyoak, 1978; Moyer & Landauer, 1967). All of these studies have shown that as the magnitudes between the compared numbers decrease in distance, the number of errors increases and response times increase. This pattern has been referred to as the *distance effect.* This finding supports the idea of a mental number line because the likelihood of errors and speed of response times is consistent with the distance of the relative magnitudes in their internal representations. Thus, adults are likely to make mistakes accessing the relative magnitudes of close numbers as representations of these magnitudes are closer together and harder to tell apart.

Further studies have been done to assess whether fractions also show this same distance effect. If so, this suggests that fraction magnitudes are represented on a mental number line similarly to whole numbers. The first study that examined this question by Bonato et al. (2007) concluded that adults do not represent fractions holistically as a single magnitude but use strategies that involve comparing the whole number components of fractions (numerators or denominators), thus exhibiting a whole number bias.[1] That is, adults were comparing the fraction values by using their whole number parts rather than the whole fraction value. Bonato et al. (2007) came to this conclusion because they found distance effects in a fraction comparison task, not for the

* Corresponding author.
E-mail address: svosniad@phs.uoa.gr (S. Vosniadou).

[1] The term 'whole number bias' refers to the application of whole number knowledge in situations where this is not appropriate (see Ni & Zhou, 2005), regardless of whether this may sometimes lead to a correct response.

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magnitudes of the fractions, but for the magnitudes of their whole number components.

However, recent experiments with adults have demonstrated that the distance effect can indeed be obtained in some fraction comparison tasks with adults (Meert, Grégoire, & Noël, 2009, 2010; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Schneider & Siegler, 2010; Sprute & Temple, 2010). Meert et al. (2009) showed that componential representations were activated when adults were asked to compare fractions with common denominators (e.g., 3/8 and 7/8) but the magnitude of the whole fractions were accessed when comparing fractions with common numerators (2/7 and 2/3). Further experiments by Meert et al. (2010) indicated access to the magnitude of the fractions in tasks where the fractions used had no common components (e.g., 5/7 and 3/8), although the variability of the congruity of the components also affected performance, suggesting that the relative magnitude of the components was being assessed, as well.

According to Schneider and Siegler (2010), the Bonato et al. (2007) results were based on fraction comparison tasks that could be quickly and accurately solved by comparing only the whole number components of the fractions, without the need to compare the fraction magnitudes. They found that when adults compare fractions that are not constrained to make whole number components straightforward predictors of the correct answers, they do rely on fraction magnitudes to solve comparison problems. In three experiments that used a range of different populations and stimuli, Schneider and Siegler (2010) found that accuracy improved and reaction times decreased as the distance between fraction pairs increased. They concluded that a representation of fraction magnitude is present in the fully developed number system of skilled adults, that "this representation resembles the mental number line used with whole numbers, and that adults use this representation to compare fraction magnitudes when simpler approaches would not yield accurate performance" (p. 1235). According to Siegler and his colleagues (Siegler, Thompson, & Schneider, 2011), the representation of fractions is the product of a continuous numerical development during which "the central conceptual structure for whole numbers, a mental number line, is eventually extended to other types of numbers, including rational numbers." (p. 274).

The purpose of the research presented in this article is to investigate in greater detail the nature of the representation of fractions in skilled adults. One possibility is that over the course of development, the conceptual structure of number expands beyond whole numbers to encompass other number types such as fractions. Assuming that this conceptual expansion is successful, the representation of number as rational number should replace the representation of whole number on the number line. In this case skilled adults should have no difficulty with simple fraction comparison tasks. Another possibility is that in the course of development adults may develop an altogether new representation of fractions, which does not replace the representation of whole number but co-exists with it. Because this new representation is however less easily assessable than the old one, adults prefer to access the whole number representation and to develop strategies that allow them to calculate the magnitude of the fraction based on the representation of whole number. In this latter case we should be able to find whole number interference even in adults with a simple fraction comparison task.

1.2. Conceptual development of fractions

The question of how fractions are represented and processed in adults is interesting and important also because it has implications about the nature of mathematical cognition and its development. As argued by Bonato et al. (2007) it is widely accepted that human numerical thinking is based on an innate, preverbal system of analog magnitudes, where numerical magnitude is represented on the continuum of real numbers, conceived as an analogical mental number line (Dehaene, Dehaene-Lambert, & Cohen, 1998; Gallistel & Gelman, 2000). If indeed numerical magnitude of any number type (i.e. fraction, decimal, integer) was represented on the continuum of real numbers, then the processing of fractions should pose no challenge, as it would provide easy access to a fraction's true value.

However, there is a great deal of evidence that fraction understanding is very difficult for children and that even after considerable mathematics instruction many students fail to perform adequately even in simple fraction tasks (Hartnett & Gelman, 1998; Mack, 1995; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler et al., 2011; Smith, Soloman, & Carey, 2005; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2010). For example, only 50% of a nationally representative sample of U.S. 8th graders were found to be able to correctly order three fractions on the National Assessment of Educational Progress (Martin, Strutchens, & Elliott, 2007).

Children's difficulties with fractions have been associated with the whole number bias (Ni & Zhou, 2005). Many fraction misconceptions have their roots in children's belief that properties of whole numbers can apply to fractions. For example, children do not understand fractional notation; they often treat fractions' numerators and denominators as two separate whole numbers, believe that the value of the fraction increases when either the numerator or the denominator increase, and think that the unit is the smallest fraction (Mack, 1995; Stafylidou & Vosniadou, 2004). In operations with mixed numbers many students ignore the fractional parts focusing only on the whole numbers (Fazio & Siegler, 2011), and continue to believe that multiplication always makes the number bigger and division always makes the number smaller (Fischbein, Deri, Nello, & Marino, 1985). Many children do not understand the density and infinite divisibility of number (Smith et al., 2005). Even after many years of mathematics instruction they do not understand that fractions do not have unique successors and that there is an infinite number of numbers between them (Vamvakoussi & Vosniadou, 2010).

For these reasons, some researchers have argued that understanding rational numbers requires conceptual change (Christou & Vosniadou, 2012; Hartnett & Gelman, 1998; Smith et al., 2005; Vamvakoussi & Vosniadou, 2010; Vosniadou, 2013; Vosniadou & Verschaffel, 2004). Vosniadou and her colleagues (Vosniadou, 2013; Vosniadou et al., 2013; Vosniadou & Skopeliti, 2013; Vosniadou, Vamvakoussi, & Skopeliti, 2008), have put forward a 'framework theory' approach to conceptual change which makes the following claims regarding mathematical development: (1) During the preschool years children form an initial concept of number, which is essentially equivalent to the mathematical concept of whole number. This initial number concept encompasses a number of background assumptions and beliefs that underlie students' expectations about what counts as a number and how it is supposed to behave (e.g., that numbers have only one symbolic representation, obey the successor principle, are ordered by means of their position on the count list with longer numbers always being bigger, etc.) which need to be re-interpreted in the process of understanding rational number; (2) Rational number information coming from instruction violates basic principles of the whole number concept (e.g., regarding symbolic notation, ordering, discreteness, the operations of multiplication and division). Thus, understanding rational number requires a restructuring of the whole number concept and the construction of a new representation of rational number on the number line; (3) This process of restructuring takes a long time to be accomplished and leads to the