Learning and Instruction 37 (2015) 56–61

Contents lists available at ScienceDirect

Learning and Instruction

journal homepage: www.elsevier.com/locate/learninstruc

Variability in the natural number bias: Who, when, how, and why

Martha W. Alibali*, Pooja G. Sidney

University of Wisconsin–Madison, USA

ARTICLE INFO

Article history:
Received 14 January 2015
Accepted 21 January 2015
Available online 13 February 2015

Keywords:
Mathematics
Rational numbers
Operations
Strategy choice
Natural number bias

ABSTRACT

When reasoning about rational numbers, people sometimes incorrectly apply principles or rules for natural numbers. Many factors affect whether participants display this *natural number bias*, including their age and experience, the affordances and constraints of the given task, and even the specific numbers in the given problem. In this paper, we argue that this variability can be conceptualized in terms of dynamic choices among problem-solving strategies. People's strategy choices vary as a function of their repertoire of available strategies and as a function of the specifics of the tasks, problems, and context. Further, we argue that the specific profiles of variability in strategy use that are observed in different participant groups can be conceptualized in terms of the strength and precision of the representations of numbers and operations that people in those groups possess. In our view, the natural number bias arises when people's representations of rational number magnitudes or rational number operations are not sufficiently strongly activated or sufficiently precise to guide performance on a specific task in a specific context. In these cases, participants' more highly activated or more precise representations for natural numbers may underlie and guide their performance. This account suggests that contexts and experiences (including instructional experiences) that help build, strengthen, and activate rational number representations should lead to improvements in performance.

1. Introduction

People sometimes use principles or rules for natural numbers in situations where those principles or rules do not apply. For example, when comparing fraction magnitudes, people sometimes choose as larger the fraction that has larger whole numbers as parts; so, for example, people sometimes judge 1/4 as greater then 1/3, because 4 is greater than 3 (Behr, Wachsmuth, Post, & Lesh, 1984). Similarly, when comparing decimal magnitudes, people sometimes choose as larger the decimal that is longer; so, for example, people sometimes judge .835 as greater than .87, because 835 > 87 (Durkin & Rittle-Johnson, 2015). This tendency to over-extend natural, whole number principles and rules, particularly when reasoning about rational numbers, has been termed the *whole number bias* (Ni & Zhou, 2005), or, alternatively, the *natural number bias* (NNB).

People's behavior is inherently variable (Alibali & Sidney, in press; Siegler, 2007), and behavior that manifests the NNB is no exception. There is substantial variability in whether and how the NNB is manifested across people, tasks and contexts. Not all people display the NNB in every situation in which they could or might be expected to do so. Moreover, people sometimes display the NNB on some items for a given task, but not on others.

In this commentary, we first consider variability in the NNB. Put simply, *who* displays the NNB, and *when* do they do so? Next, we seek to explain this variability in terms of *how* people solve problems that involve rational numbers. Finally, we consider *why* people manifest the specific profile of variability that we observe. We argue that the nature of the observed variability depends on the representations that are most highly activated for a given person and in a given context. We close by considering the implications of this view for future research.

2. Variability in the natural number bias: who shows it, and when?

Who displays the natural number bias? It seem that everyone does, in some form or another. The NNB been observed in elementary school students (e.g., Meert, Grégoire, & Noël, 2010), in high school students (e.g., DeWolf & Vosniadou, 2015), in adults (e.g., Vamvakoussi, Van Dooren, & Verschaffel, 2012b), and even in expert mathematicians (e.g., Obersteiner, Van Dooren, Van Hoof, &

* Corresponding author. Department of Psychology, University of Wisconsin–Madison, 1202 W. Johnson St., Madison, WI 53706, USA.
E-mail address: mwalibali@wisc.edu (M.W. Alibali).

http://dx.doi.org/10.1016/j.learninstruc.2015.01.003
0959-4752/

Verschaffel, 2013). Thus, the bias is evident, not only in learners who have just been introduced to rational numbers, but also in individuals who have extensive familiarity with rational numbers. However, the pattern of tasks, contexts, and response measures on which the NNB is displayed varies across participant groups. In some cases, people display the NNB in their patterns of errors; in other cases, in reaction times; and in still other cases, in the strategies that they report using to solve problems. In our view, this variability is potentially informative about the processes that give rise to the NNB.

When do people display the NNB? That is, what sorts of tasks elicit thinking based on natural numbers? One commonly used task is fraction magnitude comparison. In such tasks, people are asked either to choose the larger (or smaller) of two fractions (e.g., Behr et al., 1984; Kallai & Tzelgov, 2009), to judge whether a statement about relative fraction magnitudes (e.g., $7/8 > 2/3$) is true or false (e.g., Bonato, Fabbri, Umiltà, & Zorzi, 2007; Vamvakoussi et al., 2012b), or to compare the magnitude of a given fraction to a "standard" value (e.g., is 4/9 more or less than 3/5?) (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014; Schneider & Siegler, 2010). Response patterns for different types of comparisons indicate that participants sometimes rely on knowledge of the magnitudes of the component natural numbers (i.e., the numerators and denominators), rather than on the magnitudes of the fractions themselves. However, participants who display the NNB in fraction magnitude comparisons do not do so on every trial. For example, DeWolf and Vosniadou (2015) report that on some trials, adults rely on strategies based on natural number parts, leaving them susceptible to the NNB, but on other trials, they appear to rely on magnitude representations for fractions, and therefore do not display the NNB.

People also display the NNB when comparing decimal magnitudes, for example, when asked to identify which of two decimals is greater, or when asked to identify decimals that are "worth the same amount" as a target decimal (e.g., "Circle all the numbers that are worth the same amount as 0.51: 0.5100, 0.051, 0.510, 51"; Durkin & Rittle-Johnson, 2015). On items such as these, participants often respond in ways that reveal that they view decimals as having certain properties of natural numbers (e.g., longer numbers are greater, adding a zero to the end of a number increases its magnitude). However, participants do not display NNB-based responses on every trial. For example, Durkin and Rittle-Johnson reported that, on a pretest, 9–11 year old participants displayed the NNB on about half of the items on which they might be expected to do so.

Another task in which people display the NNB is in interpreting algebraic expressions that involve operations (e.g., Vamvakoussi, Van Dooren, & Verschaffel, 2012a). Van Hoof and colleagues (Van Hoof, Vandewalle, Verschaffel, & Van Dooren, 2015) asked high school students to judge whether certain expressions (e.g., $x < x + 2$, $x > x * 2$, $3 < 3/x$) could or could not be true. They found that students almost always considered natural number properties when evaluating the algebraic expressions (i.e., by invoking principles about natural numbers, such as "multiplication makes larger" or by replacing the unknown with a natural number), regardless of the operation. In contrast, students considered other kinds of numbers (e.g., negative whole numbers) only when evaluating expressions with addition and subtraction operations, and not when evaluating expressions with multiplication and division operations. Thus, students applied NNB-based approaches more broadly than other approaches.

3. How does the NNB arise? A dynamic strategy choice account

Some researchers have argued that the NNB arises as a consequence of a dual processing system for magnitudes (e.g., Vamvakoussi, et al., 2012b). According to this view, people engage in both *intuitive* processing, in which knowledge about numerical magnitudes is immediately and automatically activated, and therefore guides performance, and in *analytic* processing, in which the magnitudes of component parts are accessed and combined in an explicit, effortful (though sometimes error-prone) way. From this perspective, the key distinction is between a fast, automatic system and a slower, effort-driven system for processing magnitudes. Errors that manifest the NNB occur when (1) natural number magnitudes are quickly and automatically activated, and (2) effortful, analytic processing does not override those automatically activated representations. Thus, "errors may be attributed to S1's [the intuitive system's] pervasiveness and S2's [the analytic system's] failure to intervene" (Vamvakoussi et al., 2012b, p. 347).

The dual-processing account provides an apt explanation for patterns of reaction times and errors on magnitude comparison tasks. However, it fares less well in explaining patterns of responses among students asked to interpret algebraic expressions (e.g., Van Hoof et al., 2015). Qualitative data suggest that in interpreting algebraic expressions, students apply a range of explicit strategies informed by natural number knowledge.

Building on this prior work, we propose that it may be useful to incorporate both intuitive and analytic processes in a *dynamic strategy choice* account. This account acknowledges the role of intuitive processes while focusing in greater detail on the nature of people's effortful processing. Our proposal emphasizes (1) variability in people's effortful approaches to solving problems (an issue that has received relatively little attention in previous work), and (2) systematic patterns of strategy use—including strategies based on intuitive processes and those based on effortful processes—across problems, contexts, and individuals. Importantly, we do not believe that all cases of NNB stem from intuitive processes (although some do), nor that all cases of effortful processing yield responses free of NNB (although some do).

We argue that people's strategy choices depend on their repertoires of available strategies. Further, drawing on concepts from dynamic systems theory (Smith & Thelen, 2003), we argue that an individual's strategy choices also depend on the affordances of particular problems, tasks, or contexts, and on that individual's history of solving similar and related problems. These factors combine to influence behavior (including strategy choices, reaction times, and errors) in the moment. Thus, patterns of strategy use, errors, and reaction times should reveal variability, but there should also be systematicity within that variability. From this perspective, we argue that responses that manifest the NNB arise as a result of people selecting problem-solving strategies based on natural numbers in certain types of problems or in certain contexts.

In our view, one class of strategies for solving magnitude comparison problems is based on intuitive processes (akin to *retrieval* as a strategy for solving arithmetic problems; Siegler & Shipley, 1995). Specifically (and cast in strategy choice terms), one possible strategy for solving fraction magnitude comparison problems is simply to compare magnitude representations that are automatically activated (i.e., by the intuitive processing system). For example, in comparing the magnitudes of ½ and ¼, people may "just know" that ½ is greater than ¼ — or that 4 is greater than 2—and they may choose to respond on the basis of this highly activated, intuitive knowledge, whether it be knowledge of fraction magnitudes or whole number magnitudes. Thus, intuitive processing can yield responses that manifest the NNB, or responses that do not.

Another class of strategies for solving magnitude comparison problems involves more effortful strategies (akin to "*back-up*" strategies for solving arithmetic problems; Siegler & Shipley, 1995). People may reach an answer by taking a series of steps that involve encoding and operating on specific features of the problems. For