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Delayed benefits of learning elementary algebraic transformations through contrasted comparisons

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A R T I C L E I N F O

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A B S T R A C T

Students studying algebra often make mistakes because of superficial similarities between addition and multiplication problems. In two experiments, we investigated whether these errors can be prevented by presenting addition and multiplication problems in such a way that students are encouraged to compare the problems at a deeper level. In Experiment 1, 72 sixth graders were assigned to two self-learning programs. In the contrast program, addition and multiplication were mixed and juxtaposed. In the sequential program, students first received only addition problems followed by multiplication problems. The results revealed that during the training, students performed worse under the contrast condition. However, in the follow-up tests (1-day, 1-week, 3-months), these findings were reversed: the contrast group clearly outperformed the sequential group. The findings were replicated under improved methodological conditions in Experiment 2 with 154 sixth graders. These experiments show that contrasted comparison of superficially similar but conceptually different material results in improved long-term learning.

1. Introduction

Research has provided strong empirical support for comparison as a fundamental learning mechanism that has a positive effect on meaningful conceptual learning in different areas (for an overview, see Gentner, 2010; Holyoak, 2005; Rittle-Johnson & Star, 2011). When two units (e.g., objects, problems, pictures) are juxtaposed, intentional comparison promotes a deep processing of the materials because their similarities and differences become highlighted. This phenomenon helps learners to abstract principles that may be used to solve novel problems (Catrambone & Holyoak, 1989; Gentner, 1983; Gick & Holyoak, 1983). In this way, comparisons were used to learn complex concepts by being presented two examples of the same concept that differed in their surfaces. Learners who studied two problems simultaneously outperformed learners who studied the two problems separately. This outperformance has for instance been demonstrated for negotiation principles using two different cover stories for the same negotiation strategy (Gentner, Loewenstein, & Thompson, 2003) or for the concept of heat flow using two different scenarios depicting heat flow (Kurtz, Miao, & Gentner, 2001). The direct comparison of superficially different but structurally equal (i.e., isomorphic) examples appears to help learners overcome contextual limitations, a crucial step in understanding complex concepts. The instruction to compare material offers learners a way to actively construct meaningful knowledge.

Positive effects of comparisons were also demonstrated for learning mathematical procedures in real-life school settings, e.g., when learning how to solve equations (Rittle-Johnson & Star, 2007) and for computational estimation (Star & Rittle-Johnson, 2009). Comparing two solution strategies led to greater learning gains compared to the sequential processing of these strategies. Chase, Shemwell, and Schwartz (2010) showed that learners who had to compare three cases of electromagnetic flux outperformed learners who studied the cases separately. Other research has shown beneficial effects of comparing examples of the same concept. According to Hattikudur and Alibali (2010), comparing equal signs with other relational symbols is more effective than simple instruction about the equal sign. Inventing the physical formula for density using contrasted cases was superior to being told the formula, followed by practicing with the cases (Schwartz, Chase, Oppezzo, & Chin, 2011). Moreover, it has been shown that comparing incorrect examples can facilitate learning (Durkin & Rittle-Johnson, 2012).

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Although comparing two or three examples of the same concept has been shown to be beneficial for the meaningful learning of various school topics, this method is rarely used in classrooms, as for instance, in mathematics, where a great number of concepts and procedures must be learned and distinguished from each other. Therefore, we wish to examine whether the application of comparisons may be extended beyond learning single complex concepts to learning a broad range of concepts, using algebra as an example.

1.1. The challenge to acquire algebraic language competence

Mathematical competencies grow through acquiring knowledge of concepts and procedures which, over the years and under favorable conditions, build on each other and form a network of knowledge that is broadly applicable and transferable (Schneider & Stern, 2009; Stern, 1997). One challenge in learning mathematics is that the concepts are often very similar and highly related, e.g., in algebra, where students commonly have substantial difficulties learning the rule system (Blume & Heckman, 2000; Kamii & Dominick, 1997; Kieran, 1992).

Learning algebra requires not only making use of the formal language in rich contextual settings but also learning the language itself with its rules and conventions (Kieran, 2004; Kirshner & Awtry, 2004). In fact, a thorough knowledge of algebraic language provides a solid foundation and thus may be used flexibly in later applications, e.g., to successfully solve equations. Learning the language is at the core of elementary algebra. Algebraic language is best learned through frequent transformations of algebraic expressions, but this key practice is often neglected (Ottmar, Landy, & Goldstone, 2012). A common problem in managing algebraic expressions flexibly is confusion between the two dominant operations of addition and multiplication, e.g., incorrectly solving the problem "$a^2 + a^2$" as "a^4" instead of "$2a^2$".

Many authors have documented confusion errors occurring in basic transformations as well as in handling fractions and solving equations (Booth, 1988; Hirsch & Goodman, 2006; Kirshner & Awtry, 2004). In algebraic addition, like terms are summarized, e.g., "$x + x = 2x$". However, x and xy are different types and thus cannot be simplified by transformation; instead, they must be written "$x + xy$". In algebraic multiplication, like factors are summarized to exponents, e.g., "$x \cdot x \cdot x = x^3$", and unlike factors are joined to form a product, e.g., "$x \cdot y \cdot z = xyz$". Therefore, by transforming expressions, the continuity of terms in addition ($xy + x + xy + x = x + x + xy + xy = 2x + 2xy$) must be clearly distinguished from the splitting of factors in multiplication ($xy \cdot x \cdot xy \cdot x = x \cdot y \cdot x \cdot x \cdot y \cdot x = x^4y^2$). As core operations in algebraic transformations, addition and multiplication must be understood thoroughly. Instruction should clearly distinguish between these two structurally different and perceptually similar principles from the very beginning to prevent confusion and to offer a strong start in learning algebra.

Two reasons for frequent confusion between addition and multiplication in solving algebra problems are the tendency to focus on perceptual features of the problem and the tendency to hastily automate mathematical procedures. We expect systematic comparison to offer a means of overcoming both these misleading tracks.

1.2. Comparisons as a means of overcoming the tendency to focus on perceptual features

It is well documented that, when initially faced with new information, people tend to pay more attention to the salient perceptual features instead of noticing the more important structural characteristics (Chi, Feltovich, & Glaser, 1981; Gentner & Namy, 1999; Loewenstein & Gentner, 2001). In algebra, problems often appear very similar because they are composed of only letters, numbers, and signs. This similarity may induce misperceptions of the forms of the correct rules and mislead the learner's understanding of algebra rules (Kirshner & Awtry, 2004; Ottmar et al., 2012). Goldstone, Landy, and Son (2010) describe this confusion as a conflict between rule-based and perceptual processes. Without redirecting the learner's attention from the surface features to the crucial structural elements, confusion between addition and multiplication is preprogrammed. Hence, when teaching algebra, it is necessary to train students explicitly to focus on the syntactic structure, i.e., to make them recognize rules and procedures (Kirshner & Awtry, 2004; Ottmar et al., 2012).

Comparisons help to overcome this misleading tendency to focus on surface patterns (Chi et al., 1981; Gentner & Namy, 1999; Holyoak, 2005). Providing learners with the explicit instruction to compare objects may shift the focus from the surface to the deeper structural level, which results in a significant effort to detect and learn the underlying principles (Catrambone & Holyoak, 1989; Mason, 2004; Schwartz & Bransford, 1998); therefore, choosing the appropriate comparison material is a crucial factor. Depending on the presented material, certain features of the juxtaposed examples are accentuated. Winston (1975) introduced "near miss" for concepts that only differ in a small number of features. Such near-miss contrasts enhance the principle extracting and appear to be more resistant to interference effects, likely because critical convergence features are highlighted (Gick & Paterson, 1992).

Two examples may be more or less similar on the surface or structural level. During the comparison process, even the surface similarities may help to identify structural differences more easily because they make the differences more salient, which again promotes principle learning and transfer (Loewenstein & Gentner, 2001; Markman & Gentner, 1993). However, if surfaces are too dissimilar, children and even adults may miss the underlying concepts that the examples are supposed to demonstrate. This may be a problem if the learners do not have enough prior knowledge to align the examples (Gentner, 2010). The best alignment is enabled when the examples under comparison are similar both in their surfaces and in their relational structure (e.g., Richland, Morrison, & Holyoak, 2006). Thus, Gentner (2010, p. 769) suggests that "sequences of close, highly alignable exemplars should be the ideal learning situation". Inspired by this idea of repeatedly offering comparisons of similar materials, we wanted to examine whether the application of comparisons may be extended beyond learning single complex concepts to learning a broader range of principles. This might be especially promising in mathematics, where a great number of concepts and procedures must be learned and distinguished from each other.

1.3. Comparisons as a means to prevent learners from hastily automate mathematical procedures

Learning in many academic domains (particularly in mathematics) is often limited to purely superficial procedure memorization (Kamii & Dominick, 1997; NCTM, 2000). When students are repeatedly presented with similar types of problems, they may automate the solution procedure. When trying to solve new problems, a student may automatically retrieve an incorrect procedure because the problem resembles a different type of problem. An example of blind adherence to an automated procedure is illustrated when algebraic multiplication problems are incorrectly solved by adding the problem components instead of multiplying them. Students who repeatedly practiced addition rules such as "$xy + xy + xy = 3xy$" are prone to retrieve and apply this rule when