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Actions speak louder with words: The roles of action and pedagogical language for grounding mathematical proof

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ABSTRACT

Theories of grounded and embodied cognition posit that situated actions are central constituents in cognitive processes. We investigate whether grounding actions influence reasoning, and how pedagogical language influences the action–cognition relationship. Undergraduate students ($N = 120$) generated proofs for two mathematical tasks after performing either grounding or non-grounding actions. Grounding actions facilitated key mathematical insights for both tasks, but did not lead to superior proofs. Pedagogical language in the form of prompts (prospective statements) and hints (retrospective statements) accompanying grounding actions enhanced proof performance on one task but not the other. Results from transfer tasks suggested that participants learned to apply their mathematical insights to new contexts. The findings suggest that relations between action and cognition are reciprocal: actions facilitate insight, while pedagogical language strengthens the influence of task-relevant actions for proof production. Pedagogically supported grounding actions offer alternative ways of fostering mathematical reasoning.

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Can performing body-based actions help learners to reason mathematically? Mathematical proof is an area of generalized, propositional reasoning that is notoriously difficult for students (Healy & Hoyles, 2000). People may understand generalizable mathematical ideas through action, as well as language (see Nathan, 2014, for a review). We conducted an experiment in which participants were directed to perform physical actions that were either relevant or irrelevant to the solution to a mathematical proof task. We considered actions produced from both a first-person and third-person point of view. Furthermore, we examined whether pedagogical language—specifically, prospective prompts and retrospective hints verbally connecting actions to the task—provided further support for participants' proof practices. In the following section, we review the literature on grounded and embodied cognition, the impact of pedagogical language, actions performed from different points of view, and the nature of students' proof practices.

1. Theoretical framework

1.1. Grounded and embodied cognition

We place this research within the framework of *grounded* and *embodied cognition* (GEC; Barsalou, 2008; Wilson, 2002). Theories of GEC posit that the body and the world are central constituents in cognitive processes (Shapiro, 2010). The general claim of GEC is that modal-specific systems for action, perception, and introspection form the basis for cognition through situated action and simulated re-enactment of prior modal experiences. One line of evidence in support of GEC comes from studies showing that action supports insight problem solving. For example, participants directed to produce eye gaze patterns that converged to the center of a diagram were most successful at solving the General and Fortress Problem, as that pattern embodies the key insight underlying the successful solution (Thomas & Lleras, 2007). Along similar lines, participants

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who produced body-based actions during training sessions with the Tower of Hanoi integrated their motor experiences into their mental representations of the task objects (Goldin-Meadow & Beilock, 2010).

The GEC framework can be productively applied to mathematical reasoning. Although mathematical reasoning typically addresses abstract entities far removed from situated action, many basic mathematical entities are grounded in the body, spatial systems, and situated actions (e.g., Alibali & Nathan, 2012; Lakoff & Núñez, 2000; Moyer & Landauer, 1967). Body-based actions offer a potentially powerful means to ground abstract mathematical ideas in concrete form, so their meaning can be more readily understood (Goldstone & Son, 2005; Nathan, 2008). There are many examples of such *grounding actions* in mathematical reasoning, including children counting with fingers to ground concepts of number (Carpenter & Moser, 1984) and teachers representing mathematical objects (such as a cube) with hand gestures to address student misunderstandings during instruction (Alibali et al., 2013).

Of course, much of formal mathematics emerged not simply from actions and perceptions, but from the social construction of mathematical knowledge mediated by language (e.g., Hersh, 1997; Romberg & Kaput, 1999). Action and perception may underpin early math development (e.g., the approximate number system, Dehaene, Bossini & Giraux, 1993), but by themselves, action and perception are inadequate for constructing the interconnected and self-referential system that we identify as the discipline of mathematics, which includes exact mathematics, precise definitions, and careful argumentation (Núñez, 2009; Rips, Bloomfield, & Asmuth, 2008). Language seems necessary for some mathematical reasoning, and may even *enable* exact mathematics, for example, exact representations of large numbers (Spelke & Tsivkin, 2001). Thus, language, along with action, offers a system that grounds mathematical cognition (Lakoff & Núñez, 2000) and supports its development (Vygotsky, 1986).

1.2. Point of view of actions

Human actions can exhibit a first-person or third-person *point of view*. That is, actions can be produced to convey the actor's own perspective, or the perspective of an observer. For example, McNeill (1992) observed that speakers' gestures often shift between first-person (*character*) and third-person (*observer*) viewpoints. Parrill (2011) documented ways that narrators assume a first-person perspective when their body parts mapped directly onto the corresponding body parts of the character being described, but conveyed a third-person viewpoint when the speakers' hands simply traced the character's motion. Gerofsky (2010) coded middle-school students' actions on graphs of polynomial functions as third-person when they *traced* the graphs, and first-person when students configured their bodies to actually *be* parts of the graphs. When an actor assumes a third-person perspective, they enact the actions of another character or object *as if* it were being observed.

There is little consensus on how point of view in gesture and action relates to cognition. Some studies have suggested that the "close-up" perspective manifested in first-person gestures is used more often by high-performing students, and may help explain their higher levels of engagement (Gerofsky, 2010). Others have suggested that third-person viewpoint allows for superior transfer, because it strips perceptual details from the action being simulated (Goldin-Meadow & Beilock, 2010). Because actions and gestures from both viewpoints have been associated with benefits for cognition, we compared actions from each viewpoint in this study.

1.3. Pedagogical language

Language is useful in mathematics learning and instruction in a variety of ways. For example, prompts that foster language production, such as generating self-explanations, are effective for promoting mathematics argumentation and learning (e.g., Hilbert, Renkl, Kessler, & Reiss, 2008; Rittle-Johnson, 2006) and quality mathematical argumentation (Kollar, Ufer, Reichersdorfer, Vogel, Fischer & Reiss, 2014). Language is also crucial for learning through its role in *pedagogy*—the transmission of semantic information through ostensive, referential communication (Csibra & Gergely, 2009). Although "telling" has its limitations, it can be effective for imparting information and correcting misconceptions (Carroll, 1968). Language may be especially important for making key conceptual connections in complex learning environments.

Teachers often use spontaneous gestures along with pedagogical language in a manner that facilitates students' processing of difficult classroom material (Alibali & Nathan, 2012; Roth, 2001). For example, a teacher might respond to a student's inability to visualize a general pattern about cubes by producing gestures, coordinated with speech, that depict a hypothetical surface (Alibali, et al., 2013). Thus, pedagogical gestures, which manifest simulated actions and perceptual states, both complement and reinforce pedagogical language.

Teachers also use pedagogical language to help students connect ideas, both *retrospectively*—using hints to reference past events—and *prospectively*—using prompts to alert students to related future concepts and experiences. Both hints and prompts have been shown to foster cognitive processing (Anderson, Corbett, Koedinger, & Pelletier, 1995; Gick & Holyoak, 1980; Hilbert et al., 2008), but their contributions to mathematical proof practices, specifically in conjunction with body-based action, have not been well studied.

1.4. Mathematical proof and justification

Proof is "an essential component of doing, communicating, and recording mathematics" (Schoenfeld, 1994, p. 74). In the United States, the Common Core State Standards identify constructing viable arguments and critiquing the reasoning of others as critical skills across grade levels, and standards for high school geometry specifically call for students to learn to construct proofs of theorems (Common Core State Standards Initiative, 2010). Yet students consistently struggle with constructing, interpreting, and evaluating proofs (e.g., Hoyles & Healy, 2007; Knuth, 2002).

Several researchers have proposed frameworks through which students' proof practices can be understood (e.g., Miyazaki, 2000; Stylianides, 2008). According to Harel and Sowder (2005), a valid *transformational proof* has three characteristics. First, it is general—it shows the argument is true for all mathematical objects falling into the category. Second, it involves operational thought, such that an individual who is generating the proof progresses through a goal structure, anticipating the results of transformations. Finally, it involves logical inference, in that conclusions are drawn from valid premises. Although traditional notions view proof as involving a formal, written argument in mathematical notation, more recent work on justification and proof has emphasized the importance and validity of proofs that are communicated verbally (e.g., Healy & Hoyles, 2000; Stylianou, Blanton, & Knuth, 2009).

Proof production has been described in terms of two components (Harel & Sowder, 2005): *ascertaining*, in which the solver determines the key mathematical insights that make the assertions likely to be true; and *persuading*, in which the solver

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