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The association between numerical magnitude processing and mental versus algorithmic multi-digit subtraction in children

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ABSTRACT

When solving multi-digit subtraction problems, children are instructed to use different types of calculation methods, such as mental and algorithmic computation. It has been contended that these two methods differentially rely on numerical magnitude processing, an assumption that has not yet been tested empirically. We therefore aimed to examine the association between the ability to represent numerical magnitudes and mental and algorithmic computation. This was done by conducting a study in fourth grade children using a symbolic and a nonsymbolic numerical magnitude comparison task and two arithmetic tasks: one in which children had to use mental computation and another one in which they were required to apply algorithmic computation. Our results show that both calculation methods rely on numerical magnitude processing, however, the association is larger for mental than for algorithmic computation.

1. Introduction

The association between numerical magnitude processing and mental versus algorithmic multi-digit subtraction in children.

Children are instructed in two major methods for doing subtraction in the number domain up to 100, i.e. mental and algorithmic computation. As will be explained in greater detail below, mental arithmetic refers to performing arithmetic operations on numbers (e.g. solving $78-23 = ?$ as $78-20 = 58$, $58-3 = 55$), whereas algorithmic computation operates on digits (e.g. solving $78-23 = ?$ as $8-3 = 5$; $7-2 = 5$). In our view, there are good reasons to study the association between numerical magnitude processing and, on the other hand, arithmetic proficiency in these two domains of the elementary school mathematics curriculum. These reasons relate to the ongoing discussion on the aims and content of elementary school mathematics. Within the international mathematics education community, there are essentially two opposite movements in this debate, i.e. the traditionalists and the reformers. The traditionalists stress the importance of the development of routine expertise in school-taught standard algorithms, while the reformers focus more on the development of adaptive expertise, i.e. the disposition to solve mathematical tasks insightfully, flexibly and creatively, making use of a variety of both mental and algorithmic computation strategies (Verschaffel, Greer, & De Corte, 2007). The current research tries to add to this debate by investigating what makes children perform well in mental and algorithmic computation, and, more specifically, how both computation methods are associated with basic number sense. It has repeatedly been argued that, because of the very different nature of the cognitive processes underlying mental and algorithmic computation, they may differentially rely on basic or lower-order number sense, of which numerical magnitude processing is a major component (Berch, 2005), particularly if the written algorithms are taught in a traditional mechanistic way (Thompson, 1999; Van den Heuvel-Panhuizen, 2001; Verschaffel et al., 2007). There exists however little direct empirical evidence to support this claim. While there is an extensive amount of research on the association of children's numerical magnitude processing with their elementary mathematics achievement in general (Booth & Siegler, 2006; Bugden & Ansari, 2011; De Smedt, Verschaffel, & Ghesquière, 2009; Halberda, Mazzocco, & Feigenson, 2008; Holloway & Ansari, 2009; Kolkman, Kroesbergen, & Leseman, 2013; Sasanguie, Van den Bussche, & Reynvoet, 2012; see Chen & Li, 2014; De Smedt, Noël, Gilmore, & Ansari, 2013; Fazio, Bailey, Thompson, & Siegler, 2014; for a review), only very few studies have focused on how numerical magnitude representations are related to more specific mathematical skills. To the best of our

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knowledge, there are no empirical studies that have investigated this issue in relation to different computational methods in multi-digit arithmetic. This study therefore examined the association between children's numerical magnitude processing and their proficiency with two major methods for doing subtraction in the number domain up to 100, i.e. mental vs. algorithmic computation. In the remainder of this introduction, we describe these two computation methods, and the tasks that are most often used to assess children's numerical magnitude processing. Afterwards we present the design and research questions of the current study.

1.1. Mental versus arithmetic computation

Teaching and practicing the algorithms for the four arithmetic operations constitute a major part of the elementary school mathematics curriculum worldwide (Thompson, 1999; Van den Heuvel-Panhuizen, 2001; Verschaffel et al., 2007). These algorithms consist of a fixed sequence of well-defined and elementary calculation steps, which one has to execute on the digits in the problem. By strictly and correctly following the steps of the algorithm, a correct response is guaranteed. Typically, this algorithmic procedure is done with paper and pencil, although it is, in principle, also possible to do it mentally, as long as the number of digits in the problem is relatively small (Torbeyns & Verschaffel, 2013). Furthermore, in algorithmic computation tasks, the numbers are presented vertically (except for the division algorithm). An example of an algorithmic computation task and the steps involved in its solution, is shown in Fig. 1.

As part of the worldwide reform movement in elementary school mathematics in the 1980s, more attention is being paid at mental arithmetic, both as a step-stone to and as a valuable alternative for the written algorithms (Kilpatrick, Swafford, & Findell, 2001; Torbeyns & Verschaffel, 2013; Van den Heuvel-Panhuizen, 2001; Verschaffel et al., 2007). Mental calculation strategies are strategies whereby one calculates with the numbers in the problem (e.g. solving 78−23 = ? as 78−20 = 58, 58−3 = 55) instead of operating on the digits (e.g. solving 78−23 = ? as 8−3 = 5; 7−2 = 5). Moreover, in mental arithmetic one does not always routinely follow a single solution path, but one flexibly adapts one's solution strategy to the specific numerical features of the problem. For instance, while children may solve subtraction problems with a small subtrahend such as 72−8 = ? by directly subtracting 8 from 72 (72−2 = 70 and 70−6 = 64, so the answer is 8), they may solve problems with a large subtrahend such as 72−66 = ? by determining how much has to be added to 66 to arrive at 72 (66 + 4 = 70 and 70 + 2 = 72, so the answer is 6). The flexible use of this latter strategy, referred to as the indirect addition strategy (Verschaffel, Torbeyns, De Smedt, Peters, & Ghesquière, 2010), requires a good understanding of the numerical magnitudes in the problem and their mutual relation. Likewise, some children may flexibly solve the subtraction 73−49 = ? by first subtracting the computationally much easier number 50 from 73, and then adding 1; 73−50 + 1 = 24. Furthermore, mental arithmetic is typically done in one's head, without paper and pencil, but it may also involve written notations, e.g. when one writes down one or more partial outcomes to prevent working memory overload. Finally, in mathematics textbooks, mental arithmetic tasks are typically presented in horizontal form. Because of all these features of mental arithmetic, various mathematics educators have stated that its quintessence is that it requires children to calculate *with* their head rather than *in* their head, relying on a well-developed (lower-order) number sense and fluent knowledge of elementary number facts (Sowder, 1992; Thompson, 1999; Torbeyns & Verschaffel, 2013; Verschaffel et al., 2007).

In sum, mental computation essentially differs from algorithmic computation in that (1) the problem is solved by operating on numbers rather than on the digits, and (2) there is no single correct solution path to be followed (namely strictly following the standard algorithm). Two additional, less important, differences are (3) that the numbers in a mental calculation problem are typically presented horizontally rather than vertically and (4) that in mental calculation there is less or even no reliance on written notations. Especially because of the first two major differences, it has been argued by several mathematics educators that there will be a strong association between children's basic number sense, conceived as their ability to process numerical magnitudes, and their proficiency in mental arithmetic, whereas the association with algorithmic computation will be much smaller or even absent (Thompson, 1999; Van den Heuvel-Panhuizen, 2001; Verschaffel et al., 2007). The major goal of the present study is to subject this general claim to an empirical test.

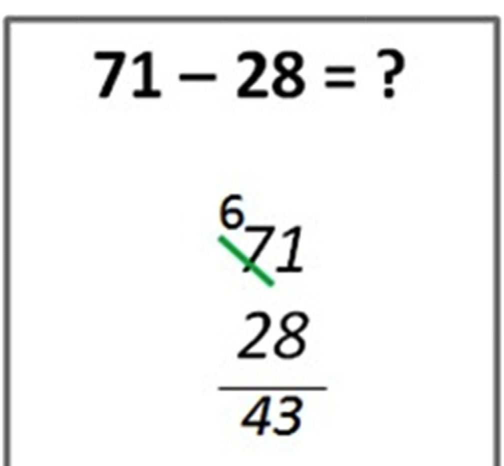

Fig. 1. An example of how the written algorithm for multi-digit subtraction is taught in Flanders (Belgium).

1.2. Numerical magnitude processing

A classic task that has been extensively used to examine numerical magnitude processing in children and adults is the numerical magnitude comparison task (Sekuler & Mierkiewicz, 1977; see also De Smedt et al. 2013). In this task, children are asked to indicate the numerically larger of two presented numerical magnitudes. Stimuli can be presented in either a symbolic (digits) or a nonsymbolic (dots) format (e.g., Holloway & Ansari, 2009) and both single-digit or double-digit stimuli can be used (e.g., Linsen, Verschaffel, Reynvoet, & De Smedt, 2014). Children's performance on this task has been shown to be associated with their general mathematics skills: Children who are faster in indicating the larger of the two numbers, show higher general mathematics achievement than children who are slower in doing so (e.g., Bugden & Ansari, 2011; Halberda et al., 2008; Holloway & Ansari, 2009; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, Van den Bussche, et al., 2012; see De Smedt et al., 2013; for a review).

The performance on the symbolic and the nonsymbolic numerical magnitude comparison task can be quantified by the speed and the accuracy by which children solved the items. Previous work with similar *symbolic* comparison tasks showed that the association between response time on the symbolic comparison task and mathematics achievement is consistent and robust (De Smedt et al., 2013; for a review). Although there are studies that have examined the accuracy by which children solve the items in the symbolic