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Subtraction by addition in children with mathematical learning disabilities

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ABSTRACT

In the last decades, strategy variability and flexibility have become major aims in mathematics education. For children with mathematical learning disabilities (MLD) it is unclear whether the same goals can and should be set. Some researchers and policy makers advise to teach MLD children only one solution strategy, others advocate stimulating the flexible use of various strategies, as for typically developing children. To contribute to this debate, we compared the use of the subtraction by addition strategy to mentally solve two-digit subtractions in children with and without MLD. We used non-verbal research methods to infer strategy use patterns, and found that both groups of children switch between the traditionally taught direct subtraction strategy and subtraction by addition, based on the relative size of the subtrahend. These findings challenge typical special education classroom practices, which only focus on the routine mastery of the direct subtraction strategy.

1. Introduction

In the last decades, variety and flexibility in children's strategy use have become major aims of mathematics education (e.g., Freudenthal, 1991; Kilpatrick, Swafford, & Findell, 2001; Verschaffel, Torbeyns, De Smedt, Luwel, & Van Dooren, 2007). To achieve these goals children are stimulated to discover and flexibly use a variety of strategies based on their understanding of number relations and/or the properties of operations. For children with mathematical learning disabilities (MLD), however, the feasibility and suitability of strategy variety and flexibility remains an issue of continued debate in many countries. Some researchers, curriculum developers, and policy makers argue that it is better for these children to develop mastery and confidence in only one way or strategy to solve problems (e.g., Geary, 2003; Milo & Ruijssenaars, 2003; National Mathematics Advisory Panel, 2008). Others claim that the development of strategy variety and flexibility should be educational goals for all students, including those with MLD (e.g., Baroody, 2003; Kilpatrick et al., 2001; Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2012; Verschaffel et al., 2007). While this discussion remains to be lively, more scientific evidence is needed.

Mental subtraction is one mathematical subdomain in which strategy variety and flexibility can be stimulated. When solving subtractions such as $81 - 43$, the most commonly taught solution strategy[1] is the *direct subtraction* strategy, in which the smaller number (43) is subtracted from the larger number (81) (e.g., $81 - 43 = (81 - 40) - 3 = 41 - 1 - 2 = 38$). However, for problems with a relatively large subtrahend compared to the difference, such as $81 - 79$, *subtraction by addition* appears to be a more clever strategy (e.g., Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). With this strategy, one can solve $81 - 79$ very efficiently by determining how much needs to be added to 79 to make 81 (e.g., $79 + 1 = 80$, $80 + 1 = 81$, so the answer is $1 + 1 = 2$). The use of the complementary addition operation on such problems can thus considerably facilitate the calculation process by reducing computational effort and increasing solution efficiency, i.e., fewer and/or smaller calculation steps, which lead faster to a correct answer (e.g., Heinze, Marschick, & Lipowsky, 2009; Verschaffel, Bryant, & Torbeyns, 2012). In contrast, for problems with a relatively small subtrahend compared to the difference, such as $81 - 2$, the subtraction by addition strategy does not lead to fewer and/or smaller calculation steps. For these problems the direct subtraction strategy seems to be more efficient.

[1] In the present study, we categorise the variety of subtraction strategies based on the main operation that is used, i.e., either subtraction or addition. Different categorisations are used by other researchers (e.g., Beishuizen, 1993; Blöte et al., 2001; Buys, 2001; Peltenburg et al., 2012), such as focussing on the manipulation of the numbers during problem solving, which leads to a classification into jump, split and varying strategies.

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Previous work on children's and adults' use of subtraction by addition in elementary subtraction indicated that children hardly use the subtraction by addition strategy spontaneously, not even on problems such as 81 − 79 (e.g., Blöte, Van der Burg, & Klein, 2001; De Smedt, Torbeyns, Stassens, Ghesquière, & Verschaffel, 2010; Heinze et al., 2009; Selter, Prediger, Nührenbörger, & Hussmann, 2012; Torbeyns, De Smedt, Ghesquière, et al., 2009). Adults, on the other hand, seem to solve symbolically presented subtractions efficiently and flexibly by means of subtraction by addition (Torbeyns, Ghesquière, & Verschaffel, 2009). These available studies relied on verbal protocol data to infer strategy use. A closer inspection of the speed data in the study by De Smedt et al. (2010) suggested that children sometimes used subtraction by addition even though they reported a direct subtraction strategy. If these children only used direct subtraction, an increase in reaction times should have been observed from items with relatively small subtrahends (81 − 7) over items with medium-sized subtrahends (81 − 43) to items with relatively large subtrahends (81 − 79), since subtracting a larger subtrahend requires more and/or larger calculation steps (Peters, De Smedt, Torbeyns, Ghesquière, & Verschaffel, 2010). This reaction time pattern was not found in De Smedt et al. (2010): Problems with a relatively large subtrahend were solved significantly faster than problems with a medium-sized subtrahend, which suggests that the actual use of the subtraction by addition strategy might be larger than revealed by the children's verbal protocols. In a recent study, Peters, De Smedt, Torbeyns, Ghesquière, and Verschaffel (2013) therefore used two non-verbal methods to infer the use of the subtraction by addition strategy in typically developing children solving symbolically presented subtractions: regression analyses and a format manipulation. They concluded that children, like adults, switched between direct subtraction and subtraction by addition to solve two-digit subtraction problems, based on the relative size of the subtrahend: The children used direct subtraction when the subtrahend was relatively small compared to the difference (as in 83 − 4), and subtraction by addition when the subtrahend was relatively large (as in 83 − 79).

So far, the use of the subtraction by addition strategy has not been explored in children with MLD, except for the study by Peltenburg et al. (2012). They showed that Dutch special education children (aged 8–12, with a mathematics level similar to the end of Grade 2) do report the use of this strategy, and this mostly on problems with a relatively large subtrahend and crossing the tens (e.g., 61 − 59); the subtraction by addition strategy was reported in more than 50% of problems of this type. These authors also found that the subtraction by addition strategy was reported more often on word problems compared to symbolically presented subtractions (i.e., 70% on adding-on word problems and 25% on taking-away word problems versus only 8% on subtractions presented in the $M - S = .$ form). While the design of Peltenburg et al.'s study included various potentially interesting numerical task features (such as size of the subtrahend, crossing the tens and closeness of minuend/subtrahend to a ten), they did not deepen the interaction between type of problem (i.e., word problems vs. symbolically presented problems) and these number characteristics (for example, large vs. medium subtrahend – they did not include problems with relatively small subtrahends). In this regard, it is important to point out that Peters et al. (2013) observed that typically developing children, when confronted with symbolically presented two-digit subtraction problems, switch between the direct subtraction strategy and the subtraction by addition strategy depending on number characteristics: Direct subtraction was used when the subtrahend was relatively small (as in 83 − 4), subtraction by addition when the subtrahend was relatively large (as in 83 − 79). Against this background, we extended the work by Peltenburg et al. (2012) in children with MLD, by investigating the role of the numbers in symbolically presented subtraction problems, also including problems with relatively small subtrahends. Furthermore, we verified whether children with MLD show similar patterns of flexible strategy use as their typically developing peers.

Moreover, it might be that the number of verbal reports of subtraction by addition on the symbolically presented problems in the study of Peltenburg et al. (2012) was an underestimation. As argued by Peters et al. (2013), children may hide the use of the subtraction by addition strategy because they think it is not valued or allowed to use other strategies to solve symbolically presented problems than the one(s) taught in the mathematics lessons (e.g., Yackel & Cobb, 1996). Furthermore, the subtraction by addition strategy can be executed very fast and quasi-automatic, and it therefore might be that children have difficulties in explaining how they found their answer: They may not have been aware of, or confused by, the steps they performed while calculating and therefore reported a strategy they knew how to explain (e.g., Cooney & Ladd, 1992; Kirk & Ashcraft, 2001). These problems might be particularly prominent in children with MLD (see Milo, 2003; Thevenot, Castel, Fanget, & Fayol, 2010). We therefore used two non-verbal methods to answer our research questions: regression analyses in which reaction times were predicted based on different task characteristics, and a method in which speed was contrasted between problems presented in different presentation formats.

2. The present study

Extending the study by Peltenburg et al. (2012), we investigated whether children with MLD switch between direct subtraction and subtraction by addition based on number characteristics when solving only symbolically presented two-digit subtraction problems, and compared their strategy use patterns with those of typically developing peers. Since verbal self-reports might be less suited to identify the subtraction by addition strategy, especially in children with MLD, two non-verbal methods were used.

First, we used the reaction times for problems presented in the standard subtraction format to calculate three linear regression models (see Peters et al., 2013; Woods, Resnick, & Groen, 1975). These models represented three different strategy use patterns. The first model, the *DS*-Model, represents the consistent use of the direct subtraction strategy. When children consistently use this strategy, the reaction times should be best predicted by the size of the subtrahend (S), because it takes longer to subtract a larger number from the minuend (e.g., 83 − 79 = .) than to subtract a smaller number (e.g., 83 − 4 = .). The second model, the *SBA*-Model, starts from the same idea but represents the consistent use of the subtraction by addition strategy: If children only use subtraction by addition, reaction times should be best predicted by the size of the difference (D), because it takes more time to determine how much needs to be added to get at a given number when the difference between both numbers is large ("How much needs to be added to 4 to have 83?") than when it is small ("How much needs to be added to 79 to have 83?"). The third model, the *Switch*-Model, represents switching between both strategies based on the relative magnitude of the subtrahend ($S < D$ vs. $S > D$), and reaction times in this model are best predicted by the minimum of subtrahend and difference ($\min[D, S]$): For problems with the subtrahend smaller than the difference (e.g., 83 − 4 = . and 84 − 38 = .), problems can be more easily solved by means of the direct subtraction strategy, and therefore reaction times for these problems are expected to increase with the size of the subtrahend. In contrast, problems with the subtrahend larger than the difference (e.g., 83 − 79 = . and 84 − 46 = .) can be more easily solved by means of the subtraction by addition strategy, and therefore reaction times for these