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How training on exact or approximate mental representations of number can enhance first-grade students' basic number processing and arithmetic skills

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A B S T R A C T

Theories of psychology and mathematics education recommend two instructional approaches to develop students' mental representations of number: The "exact" approach focuses on the development of exact representations of organized dot patterns; the "approximate" approach focuses on the approximate representation of analogue magnitudes. This study provides for the first time empirical evidence for the specific effects of these approaches by implementing them in a highly controlled learning environment. 147 first-graders were randomly assigned to one of three intervention groups that used an "exact", an "approximate", or both versions of the same computer game, or to a control group. Performance on tasks requiring exact or approximate number processing as well as achievement in arithmetic were measured before and after the intervention. Results show that performance improved on tasks related to the exact or approximate number aspect trained, but there was no crossover effect. Achievement in arithmetic increased for the experimental groups and tended to be higher after only exact or only approximate training. Implications for teaching and learning in the classroom are discussed.

1. Introduction

Children at a very young age seem to be able to mentally represent small numerosities precisely and larger numerosities approximately. Through instruction, children are supposed to further develop mental representations of numbers that are thought to be fundamental for higher-order arithmetical achievement. The two primary approaches to achieving this are (1) using tasks and external representations that emphasize the exact meaning of numbers and (2) using tasks and external representations that emphasize the approximate meaning of numbers. Both approaches are well grounded in theory and supported by some empirical evidence. However, to our knowledge, no study has directly compared the differential effects of both approaches in a controlled experimental setting. In this article, we present an experimental study where we contrasted the two approaches at the middle of grade 1, when children learn to deal with numbers up to 20. In the following, we elaborate on the mental representations of numbers, how these representations are related to further arithmetical development, and how they can be enhanced by instruction.

1.1. *Cognitive systems for numerical representation*

Evidence from infant studies (e.g., Feigenson & Carey, 2003; Feigenson, Carey, & Hauser, 2002; Feigenson & Halberda, 2004; Wynn, 1992; Xu, 2003; see Barth, Baron, Spelke, & Carey, 2009), brain imaging (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003; Kucian, Aster, von Loenneker, Dietrich, & Martin, 2008; Piazza, Mechelli, Butterworth, & Price, 2002; Pinel, Dehaene, Riviere, & LeBihan, 2001), and experimental psychology (e.g., Dehaene & Cohen, 1994; Mandler & Shebo, 1982; Sekuler & Mierkiewicz, 1977; Simon, Peterson, Patel, & Sathian, 1998) suggests that basic numerical processing is achieved through two distinct cognitive systems that represent numerical information (see Feigenson, Dehaene, & Spelke, 2004, for a review). While one system represents small numerosities precisely, the other system represents larger numerosities approximately. Representation in the "exact" system is thought to be discrete, with each object in the external world being represented by a mental entity. In the "approximate" system, numerosities are represented analogously, and the representation gets more imprecise with increasing magnitude.

During numerical development, these non-verbal representations become linked to verbal representations, such as number words, and to symbolic representations, such as Arabic numbers. According to models developed by Dehaene (1992) and Von Aster (2000), the two non-verbal representations of number are integrated in one module,

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which is responsible for the semantic understanding of numerical information. Because they are developed prior to the other modules, the two non-verbal systems are thought to be the "core systems of number" (Feigenson et al., 2004, p. 307).

1.2. Mental number representations and basic number processing

The exact cognitive number system is thought to underlie the quick recognition of small numerosities, for which the term *subitizing* has been coined (e.g., Mandler & Shebo, 1982). When individuals are asked to determine the numerosity of a set of objects, they have been found to be almost equally fast and precise for numerosities up to three or four, but response times and error rates increase rapidly for larger sets. This holds true when objects are presented in random order. For organized arrangements, performance is faster and more precise, because groups of objects can be seen as one entity representing a larger quantity ("conceptual subitizing", cf. Clements, 1999). Performance on enumeration tasks with small numbers of dots or with organized dot patterns can therefore be used as a measure of subitizing and conceptual subitizing skills, respectively.

While successfully performing these tasks is thought to rely on the exact number system (see Section 1.1), number comparison tasks have frequently been used as a measure of the approximate number system (e.g., De Smedt, Verschaffel, & Ghesquière, 2009; Sekuler & Mierkiewicz, 1977; Szücs & Goswami, 2007). In these tasks, the larger of two numerosities (non-symbolic number comparison or magnitude comparison) or the larger of two number symbols (symbolic number comparison) has to be determined. As it is not necessary (rather disadvantageous) to determine the exact difference of the numbers (or numerosities, respectively), or even to represent the numbers (resp. numerosities) exactly, number comparison is regarded to rely on the approximate number system (see Gilmore, Attridge, & Inglis, 2011). Another task relying on the approximate number system is approximate calculation (e.g., Kucian et al., 2006). A typical task in this respect is to decide whether $7 + 8$ is closer to 10 or 16.

Performance on both types of tasks has proven to be relevant for further numerical development in children. For example, Landerl, Bevan, and Butterworth (2004; see Koontz & Berch, 1996; Penner-Wilger, Fast, LeFevre, & Smith-Chant, 2007) found that eight to nine year old children with severe problems in mathematics also showed deficits in subitizing. The same seems to hold true for conceptual subitizing (Mulligan, Mitchelmore, & Prescott, 2006).

An even larger body of evidence has shown that approximate number processing is related to arithmetical achievement. In a longitudinal study, Halberda, Mazzocco, and Feigenson (2008) found that performance on a non-symbolic number comparison task at grade nine was retrospectively predictive for mathematics achievement in each year from kindergarten to grade 6. For the symbolic number comparison task, De Smedt et al. (2009) showed that performance in grade 1 was substantially correlated with performance in arithmetic in grade 2. Siegler and Opfer (2004) asked children to indicate the position of a given number symbol on an empty number line. Performance on this task was related to achievement in mathematics in kindergartners as well as in students in grade 1 and grade 2.

The assumption that performance on the abovementioned basic number processing tasks is relevant for arithmetic development is largely based on correlation or regression analysis.[1] There are only a small number of intervention studies, in which the development of mental number representations in children was enhanced and the impact on achievement in arithmetic was evaluated. Those studies differ in the instructional approach they pursue to enhance either exact or approximate mental number representations.

1.3. Two main approaches to enhance mental number representations

Following different theories about numerical development, interventions differ in the types of problems children are asked to solve and in the way numbers are externally represented. Some researchers argue that the approximate number system is most important for numerical development (e.g., Dehaene, 2009). This would imply to prefer approximate external representations of numerical information rather than exact representations. Approximate numerical information can be represented using discrete units like dots, when students are asked to estimate the numerosities of large sets of objects. A more frequently recommended approach is to use analogue representations of numbers, such as an empty number line (e.g., Beishuizen, 1999; Klein, Beishuizen, & Treffers, 1998), where students are asked to indicate the approximate position of numbers. In both cases, the relative numerical magnitude is more important than the exact one. Recognizing relative number magnitudes quickly is important for calculation, as it helps to find elaborated strategies. For instance, instead of adding 9, it may be easier to add 10 and take away 1. In order to r*ecognize* that this strategy is helpful a quick representation of the relative magnitudes of 9 and 10 is required.[2]

Other researchers argue that recognizing small numerosities (based on subitizing) is the most fundamental skill for numerical development (e.g., Clements, 1999; Butterworth, 2005). To make use of subitizing, larger sets of dots should be represented in organized patterns, so that numerosity judgements can be made through conceptual subitizing (cf. Section 1.2). Moreover, using structures of ten could motivate students to group dots in tens and therefore prepare them to understand the decimal number system. In German grade 1 classrooms, the ten-frame and the twenty-frame are popular examples for such an instructional approach. In addition to groups of ten, these external representations make use of groups of five so that conceptual subitizing can be developed progressively. The twenty-frame is considered particularly helpful when students learn to add single-digit numbers with carries over ten. The theoretical assumption is that externally representing such additions could result in mental representations with equivalent structures.

There is limited evidence for each of the abovementioned instructional approaches. Kucian et al. (2011; cf. Siegler & Ramani, 2009; Ramani & Siegler, 2011) trained preschool children with low numerical abilities using a computer game, in which the player had to navigate a landing rocket onto the correct position on a horizontal number line. The game aimed at developing a mental number line. Indeed, Kucian et al. could show that after a training phase of 15 min, five days a week, for five weeks, students performed better on arithmetical tasks. Particularly high training gains were found for low-achieving children on number line tasks. Training effects could also be found on brain activation in parietal brain areas, hinting to a qualitative change of mental processing after training and supporting the existence of an approximate cognitive number system (see Section 1.1). Wilson, Dehaene, Dubois, and Fayol (2009) also aimed at the development of children's approximate mental number representation. In their game "The Number Race" (Wilson et al., 2006), the player has to make number comparison

[1] The same holds true for the relevancy of other basic number processing tasks, such as counting (Aunio & Niemivirta, 2010; Krajewski & Schneider, 2009) or focusing on numerosity (Hannula & Lehtinen, 2005).

[2] This is of course not sufficient to solve the task completely. To do so, exact calculation is necessary as well.