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Improving students' proportional thinking using schema-based instruction

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Abstract

This study investigated the effectiveness of an instructional program (schema-based instruction, SBI) designed to teach 7th graders how to comprehend and solve proportion problems involving ratios/rates, scale drawings, and percents. The SBI program emphasized the underlying mathematical structure of problems via schematic diagrams, focused on a 4-step procedure to support and monitor problem solving, and addressed the flexible use of alternative solution strategies based on the problem situation. Blocking by teacher at three middle schools, the authors randomly assigned the 21 classrooms to one of two conditions: SBI and control. Classroom teachers provided the instruction. Results of multilevel modeling used to test for treatment effects after accounting for pretests and other characteristics (gender, ethnicity) revealed the direct effects of SBI on mathematical problem solving at posttest. However, the improved problem solving skills were not maintained a month later when SBI was no longer in effect nor did the skills transfer to solving problems in new domain-level content.

Keywords: Word problem solving; Ratio and proportion; Middle school students; Schema-based instruction

1. Introduction

Problem solving is a central focus of current educational reforms in mathematics (Australian Education Council, 1990; Cockcroft, 1982; Department of Education and Employment [DfEE], 1999; National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008; National Research Council, 2001). Foundational to problem solving in the middle grades is proportional thinking (reasoning with ratios, rates, and percentages), which requires "understanding the multiplicative relationships between rational quantities" (Boyer, Levine, & Huttenlocher, 2008, p. 1478). Proportional thinking provides the bridge between the numerical, concrete mathematics of arithmetic and symbolic algebra and higher mathematics (e.g., Fuson & Abrahamson, 2005; Lamon, 2007; Post, Behr, & Lesh, 1988). However, children and adolescents experience difficulty with proportional thinking that extends into later years (Ahl, Moore, & Dixon, 1992; Fujimura, 2001).

Proportional thinking, which is typically introduced in late elementary and middle schools in the form of word problems, is often used to teach "mathematical modeling and applied problem solving" (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005, p. 58). Word problems often involve short stories depicting relations between quantities (i.e., "The ratio of red to yellow roses in Monica's bouquet is 3:5. The bouquet has a total of 2 dozen red and yellow roses. How many red roses are in Monica's bouquet?"). Proportion word problems such as this one are complex, in part because they require students to understand the language (i.e., grammatical rules of English) and factual information (e.g., 2 dozen = 12) in the problem, identify relevant information (the ratio of the number of red roses to number of red and yellow roses) in the problem to create an adequate mental representation, and generate, execute, and monitor a solution strategy (Desoete, Roeyers, & De Clercq, 2003; Mayer, 1999).

U.S. students' consistent difficulties in this domain call for effective instructional practices. Several recommendations are

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reported in the research literature for developing children's proportional thinking, including providing ratio and proportion tasks in a wide range of contexts (e.g., measurements, prices, rates) and ensuring that students have experienced conceptual instruction before presenting symbolic strategies such as the cross-product algorithm for solving proportional problems (Lamon, 1999; Van de Walle, 2007). Furthermore, designing instructional tasks that allow certain types of peer interactions may foster the development of mathematical thinking involved in proportional reasoning (Schwarz & Linchevski, 2007). In the present randomized controlled study, we rigorously evaluated the effectiveness of one instructional practice, schema-based instruction (SBI), which has shown promise in prior work (e.g., Jitendra et al., 2009).

1.1. Research on schema-based instruction and problem solving

SBI is a promising instructional approach for improving mathematical word problem solving skills for both students at risk for poor problem solving outcomes as well as typically achieving students (e.g., Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Jitendra et al., 2009). SBI is grounded in schema theory, which is based on the notion that acquisition of the problem schema, or underlying structure of the problem, is critical to successful problem solving (Kalyuga, 2006). For example, schema theory posits that organizing problems based on structural features (e.g., *rate* problem) as opposed to surface features (e.g., the problem's cover story) can induce the solution strategy necessary for effective problem solving. Schemata are cognitive knowledge structures held in long term memory that "allow us to treat multiple elements of information in terms of larger higher-level units (or chunks)" (Kalyuga, 2006, p. 2). Although initial schema acquisition entails working memory resources, use of schemata becomes automated with sufficient practice to require minimal working memory resources (Kalyuga, 2006).

The majority of research on the use of schemata in word problem solving has been conducted in the elementary grades, where analyses have identified several types of schemata (e.g., *Change*, *Group*, *Compare*) in the domain of arithmetic word problems (see Marshall, 1995). These schemata describe the semantic relations in story problems. For example, consider the following problem: *Music Mania sold 56 CDs last week. It sold 29 fewer CDs last week than this week. How many CDs did it sell this week?* (Jitendra et al., 2007, p. 118). Learners can be cued to the *Compare* schema for this problem through the relational sentence, "*It [Music Mania] sold 29 fewer CDs last week than this week,*" which illustrates the difference in the number of CDs sold last week to the number of CDs sold this week.

The use of schematic representations is a means to not only identify the underlying structure of problems but also model thinking by making apparent the mathematical relations among quantities in the problem situations (Steele, 2005). Schematic representations that can be used to interpret and elaborate on information in the problem lead to enhanced problem solving performance (Pape & Tchoshanov, 2001). Such representations are essential components of many studies of SBI (e.g., Fuchs et al., 2009; Fuchs, Seethaler et al., 2008; Fuson & Willis, 1989; Griffin & Jitendra, 2008; Jitendra et al., 2009; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra et al., 2007; Jitendra et al., 1998; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999; Lewis, 1989; Willis & Fuson, 1988; Xin, 2008; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009; Xin, Wiles, & Lin, 2008; Zawaiza & Gerber, 1993). For example, Fuchs et al. (2008) explored the effects of SBI for third graders identified as having mathematics and reading difficulties. The 35 students, who scored at the 10th percentile in math and reading, were randomly assigned to the SBI or a control group that received regular mathematics classroom instruction. The SBI intervention in this study also focused on teaching students to transfer their word problem skills to more challenging problems that contained irrelevant information or novel questions that entailed an extra step, or relevant information presented in charts, graphs, or pictures. Results revealed that students in the SBI group improved their word problem solving performance compared to a control group. The effect size comparing the SBI group with the control group was large ($d = 1.80$).

Similarly, Jitendra and colleagues (Jitendra et al., 1998, 2007) worked with third graders who were randomly assigned to an SBI or control group. In both these studies, all students received comparable instructional time on problem solving heuristics, yet SBI was more effective than the control condition at enhancing students' mathematical word problem solving skills, regardless of whether classroom teachers delivered SBI in a whole-class format or researchers provided instruction in a small group arrangement. The effect sizes comparing the SBI group with the control group were moderate to large at immediate posttest ($d = 0.52$ to 0.65) and delayed posttest ($d = 0.69$ to 0.81). Further, SBI improved transfer to novel problems and the state standardized assessment ($d = 0.65$ to 0.74).

While the benefits of schema-based instruction are well-established with young children working on addition/subtraction word problem solving, less is known about the effectiveness of this approach in the middle grades with proportion word problems. In fact, given the substantial differences in the research on (and the content of) arithmetic vs. proportion word problems, there is some question as to whether SBI will be as effective in the middle grades. More specifically, there are three features of the arithmetic word problem landscape that have played key roles in the success of SBI - none of which applies to proportion word problems.

First, there is an extensive and mature literature in mathematics education that has identified a small set of agreed-upon arithmetic word problem types that completely characterize the domain. Carpenter and colleagues (Carpenter, Hiebert, & Moser, 1981; Carpenter & Moser, 1982) are generally credited with establishing the problem typology in this domain by identifying four problem types: *Change*, *Combine*, *Compare*, and *Equalize*. Across numerous studies in education and psychology (including existing research on SBI), researchers universally use this typology. Second and related, among SBI researchers, there are agreed-upon and very similar schemata