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The effects of different modes of representation on the solution of one-step additive problems

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Abstract

This study investigated the role of different modes of representation, i.e., verbal description, decorative pictures, informational pictures and number line, in solving additive change problems. Data were collected from 1447 students in Grades 1, 2, and 3. Structural equations modelling affirmed the existence of four first-order representation-specific factors indicating the differential effects of the representations and a second-order factor representing the general mathematical problem-solving ability. It also provided support for the invariance of this structure across the three age groups. Rating scale analysis showed the interaction of the representational affiliation and the mathematical structure of the problems in their hierarchical ordering.

Keywords: Additive problems; Representations; Verbal description; Decorative pictures; Informational pictures; Number line; Structural equations modelling

1. Introduction

In mathematics education, visual representations are important both as an aid supporting reflection and as a means for the communication of mathematical ideas. Therefore, researchers believe that visual representations are an important aid for learning (DeLoache, 1991) and problem solving (DeWindt-King & Goldin, 2003; Diezmann & English, 2001). This study aims to shed light on the influence of three types of representation on additive problems. Specifically, we investigate the role of informational pictures, decorative pictures, and the use of the number line. These are contrasted to each other and to the use of plain verbal description (written text) for the solution of one-step addition and subtraction problems presented in a number of different structures to be described below. Specifically, below we first discuss the nature and possible effects of different types of representation of arithmetic problems and then the different structures in which these problems may be presented.

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1.1. Representations in mathematics learning

A representation is any configuration of characters, images or concrete objects that stand for something else (DeWindt-King & Goldin, 2003; Goldin, 1998; Kaput, 1985). Schnotz (2002) suggests that text and visual displays belong to different classes of representations, namely descriptive and depictive representations, respectively. Descriptive representations consist of symbols that have an arbitrary structure and are associated with the content they represent simply by means of a convention. Depictive representations include iconic signs that are associated with the content they represent through common structural features on either a concrete or a more abstract level.

Given that a representation cannot describe fully a mathematical construct and that each representation has different advantages, using multiple representations for the same mathematical situation is at the core of mathematical understanding (Duval, 2002). Ainsworth, Bibby, and Wood (1997) suggest that the use of multiple representations can help students develop different ideas and processes, constrain meanings and promote deeper understanding. By combining representations students are no longer limited by the strengths and weaknesses of one particular representation. For example, we use pictures in mathematics textbooks to increase the "readability" of standard mathematical expressions. However, interacting with multiple representations requires the understanding of the relationship between them. This is a complex process. Research shows that students encounter difficulty in integrating information from different sources (Case & Okamoto, 1996; Demetriou, Christou, Spanoudis, & Platsidou, 2002) or in moving from one representation of a mathematical object to another. As a result, they tend to use representations in isolation (Ainsworth, 2006; Duval, 2002).

Carney and Levin (2002) proposed five functions that pictures serve in text processing: decorative, representational, organizational, interpretational, and transformational. Decorative pictures simply decorate the page, bearing little or no relationship to the text content. Representational pictures illustrate a part or all of the text content. Organizational pictures provide a useful structural framework for the text content. Interpretational pictures help to clarify a difficult text. Finally, transformational pictures include mnemonic components that are designed to improve recall of information by the thinker.

However, the use of pictorial representations may not have the intended effects due to obstacles they may cause to mathematics learning and problem solving (Bishop, 1989). For instance, these representations may divert attention to irrelevant details and they may highlight some aspects of the problem at the expense of others, more relevant to the task requirements (Colin, Chauvet, & Viennot, 2002; Presmeg, 1986). Moreover, a pictorial representation may fail to help in an educational setting, such as mathematical problem solving, when students do not understand how the representation is related to its referent (DeLoache, Uttal, & Pierroutsakos, 1998).

In elementary mathematics, the number line is a representation that is widely used for the teaching of basic whole number operations and arithmetic in general (Klein, Beishuisen, & Treffers, 1998). Case and Okamoto (1996) suggest that six-year-olds and older children know and consistently use a linear structure, which is depicted as a number line with linearly increasing quantities, for representing numbers. Constructing a linear representation of numbers enables children to solve various mathematical problems that they could not tackle before. Siegler and Booth (2005) provided further support for the importance of the linear pattern of representing numbers to mathematical development in the early elementary school years. They examined the development of numerical estimation in young children and the internal representations that yielded the estimates. An important finding was that estimation accuracy was improved as a result of the increasing reliance on linear representations. This was supported also by Siegler and Opfer's (2003) findings on children and adults' performance while estimating the placement of numbers on number line. Siegler and Booth (2005) suggest that the number line may be used as a teaching tool for promoting reliance on linear representations and for helping children understand the meanings of any range of numbers.

Gagatsis, Shiakalli, and Panaoura (2003) consider the number line as a geometrical model, which involves a continuous interchange between a geometrical and an arithmetic representation. From the point of view of the geometric dimension, the numbers depicted on the line correspond to vectors and to the set of discrete points of the line. From the point of view of the arithmetic dimension, points on the line are numbered so that measuring the distance between the points represents the difference between the corresponding numbers. Despite the widespread use of the number line as an aid to operations on numbers, doubts about its usefulness have been raised (Hart, 1981). Ernest (1985) maintains that there can be a mismatch between students' understanding of whole number addition and their understanding of the number line model of this operation. The simultaneous presence of the geometric and the arithmetic conceptualisation of number may limit the effectiveness of the number line and thus hinder the performance of students in arithmetical tasks (Gagatsis et al., 2003).