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Nodal S_N-method for *HEX-Z* geometry

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Abstract

The problem of spatial approximation becomes very important in the solution of neutronics problems with coarse spatial grids, in particular, in the calculations of fuel assemblies of fast reactors (for instance, BN-800 and BN-1200 reactors) with computational cell in the form of hexagonal prism.

"Weighted diamond difference" (WDD) schemes are the most widely used among the finite difference schemes for the neutron and gamma-ray transport equation solution. They are efficient from the viewpoint of ease of their implementation and associated CPU time expenditures. However, some drawbacks of these schemes are manifested when they are applied to solve the above described problems. Diamond difference scheme (DD) having second-order approximation (the best for this class of schemes) does not possess the properties of positivity and monotonicity. This is the reason why negative values and non-physical oscillations are often present in the solutions obtained. "Step" scheme (St), which is free from the disadvantages of the diamond difference scheme, has accuracy of only the first order. In connection with the need in high-accuracy calculations its use appears to be inefficient.

There exist algorithms for correction of negative values, as well as adaptive (*AWDD*) schemes aimed both at the reduction of the level of oscillations and at the obtaining positive solutions. However, these algorithms negatively affect the order of approximation, and schemes of the first – second order of accuracy are discussed in such cases. Besides that, for adaptive schemes there exists the problem of correct selection of parameters of the scheme.

The evident way to escape such situation with simultaneously enhancing quality and accuracy of the calculation is to select a fine mesh. In case of calculation of fuel assemblies of fast reactors spatial grid represents an arrangement of rectangular prisms with regular hexagons forming their bases (in such cases reference is made to *HEX-Z*-geometry). Therefore, hexagonal cells can be divided into rhomb-shaped cells (three rhombs per one hexagon; 12 rhombs per one hexagon, etc.). Diamond scheme is applied for the grids consisting of rhombs thus obtained. Because of the smaller cell size as compared with original cell size, the drawbacks inherent to this scheme will not be pronounced. Triangular grid can also be used.

A different approach for the solution of the above indicated problem is to develop computational methods with enhanced order of accuracy without increasing the number of computational points. Nodal method is one of such methods. Expansion of unknown function inside the node (elementary volume with constant properties) in basis functions with subsequent calculation of expansion moments constitutes the basis of any nodal method.

Nodal S_N-method in *HEX-Z* geometry will be discussed in the present paper.

Keywords: Fast neutron reactor; Neutronics calculations; Nodal method; S_N-approximation; Hexagonal geometry.

Nodal S_N-method

Let us examine stationary neutron transport process described by linear Boltzmann equation. After angular and energy discretization we obtain:

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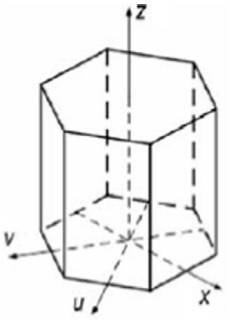

Fig. 1. Computational cell in *HEX-Z*-geometry.

$$\Omega_m \nabla \varphi_m^g(x,y,z) + \sum_t^g (x,y,z)\varphi_m^g(x,y,z) = \sum_{g'=1}^{G} \left(\Sigma_s^{g' \to g} + \frac{\chi^g}{K_{eff}} \nu \Sigma_f^{g'} \right) \sum_{m=1}^{M} w_m \varphi_m^{g'}(x,y,z), \quad (1)$$

where $\varphi_m^g(x,y,z)$ is the neutron flux density in point (x,y,z) in the direction Ω_m in group g; $\sum_t^g(x,y,z)$ is the total macroscopic cross-section of the interaction; $\Sigma_s^{g' \to g}$ is the macroscopic cross-section of scattering from group g' into group g; χ^g is the fission neutron spectrum; K_{eff} is the effective neutron multiplication factor; $\nu \Sigma_f^{g'}$ is the number of fission neutrons emitted in one fission act; w_m is the weight of the angular quadrature. Indices g and m will be hereinafter omitted by us when possible.

Let us examine the computational cell with width across flats Δx and height Δz_k as shown in Fig. 1 simulating one of the altitudinal sections of fuel assembly of fast reactor core:

$$D = \left\{ \left[-\frac{\Delta x}{2}, \frac{\Delta x}{2} \right] \times [-y_b(x), y_b(x)] \times [z_{k-1/2}, z_{k+1/2}] \right\},$$

$$y_b(x) = \frac{\Delta x - |x|}{\sqrt{3}}.$$

Let us choose for the sake of certainty angular direction $\Omega_m > 0$. Integration of Eq. (1) on variables y and z within the limits of the cell results in the following one-dimensional equation:

$$\mu \frac{d}{dx} y_b(x)\Phi(x) + \Sigma_t y_b(x)\Phi(x) = y_b(x)Q(x) - L(x) \quad (2)$$

For one-dimensional flux $\Phi(x)$:

$$\Phi(x) = \frac{1}{2y_b(x)\Delta z_k} \int_{z_{k-1/2}}^{z_{k+1/2}} dz \int_{-y_b(x)}^{y_b(x)} \varphi_m^g(x,y,z)dy.$$

$L(x)$ in Eq. (2) is the neutron leakage having radial component $L_r(x)$ and axial component $L_z(x)$ combined in (1) as follows [1]:

$$L(x) = L_r(x) + y_b(x)L_z(x)$$

$$L_r(x) = \begin{cases} \frac{\mu_u \Phi_{u+}(x) - \mu_v \Phi_{v+}(x)}{\sqrt{3}}, & x > 0 \\ \frac{\mu_v \Phi_{v+}(x) - \mu_u \Phi_{u-}(x)}{\sqrt{3}}, & x < 0 \end{cases};$$

$$L_z(x) = \xi[\Phi_{z+}(x) - \Phi_{z-}(x)]/\Delta z_k.$$

Taking into account the boundary condition $\Phi(\Delta x/2) = \Phi_{in}^x$ we obtain the followig solution of Eq. (2):

$$y_b(x)\Phi(x) = \frac{\Delta x}{2\sqrt{3}} \exp\left\{ -\frac{\Sigma_t}{\mu_x} \left(\frac{\Delta x}{2} + x \right) \right\} \Phi_{in}^x + \frac{1}{\mu_x} \int_{-\frac{\Delta x}{2}}^{x} [y_b(t)Q(t) - L(t)] \times \exp\left\{ -\frac{\Sigma_t}{\mu_x}(x-t) \right\} dt. \quad (3)$$

Now let us use polynomial expansion of neutron flux and the source as follows:

$$\Phi(x) = \sum_{i=0}^{I} \Phi_{in}^x h_i(x), \; Q(x) = \sum_{i=0}^{I} Q_i^x h_i(x),$$

$$h(x) = \left\{ 1, x, x^2 - \frac{5}{12}, \ldots \right\} : \int_{-\frac{\Delta x}{2}}^{\frac{\Delta x}{2}} y_b(x) h_i(x) h_j(x) dx = \delta_{ij} D_j^x. \quad (4)$$

Substitution of (4) in (3) allows obtaining expression for expansion moments and substitution of $x = \Delta x/2$ in (3) gives expression for flux Φ_{out}^x on the outcoming plane.

Applying similar technique we obtain expressions for functions $\Phi(u)$ and $\Phi(v)$ where variables u and v correspond to the coordinate axes shown in Fig. 1.

When coordinate system is rotated by the angle α transfer equation in new coordinates (x',y',z') is reduced to the following form:

$$\left(\mu' \frac{d}{dx'} + \eta' \frac{d}{dy'} + \xi' \frac{d}{dz'} + \sum_t \right) \varphi(x',y',z') = Q(x',y',z'),$$

i.e. to Eq. (1) with following new direction cosines:

$$\mu' = \mu \cos \alpha + \eta \sin \alpha,$$
$$\eta' = -\mu \sin \alpha + \eta \cos \alpha.$$

Thus, it is sufficient to make substitution $(\mu,\eta) \to (\mu',\eta')$ in expressions (3) and (4) for $\alpha_u = \pi/3$ and $\alpha_v = 2\pi/3$.

For variable z we have:

$$\frac{\xi}{\Delta z} \frac{d}{dz} \Phi(z) + \Sigma_t \Phi(z) = Q(z) - L(z), \quad (5)$$

$$\Phi(z) = \frac{2}{\sqrt{3}\Delta x^2} \int_{-\frac{\Delta x}{2}}^{\frac{\Delta x}{2}} dx \int_{-y_b(x)}^{y_b(x)} \varphi_m^g(x,y,z)dy,$$

$$L(z) = \frac{2}{3\Delta x} \sum_{\alpha=\{x,u,v\}} \mu_\alpha [\Phi_{\alpha+}(z) - \Phi_{\alpha-}(z)],$$

$$\Phi(z) = \sum_{i=0}^{I} \Phi_i^z f_i(z), \; Q(z) = \sum_{i=0}^{I} Q_i^z f_i(z), \quad (6)$$

$$f(z) = \left\{ 1, z, z^2 - \frac{1}{12}, \ldots \right\} : \int_{z_{k-1/2}}^{z_{k+1/2}} f_i(z) f_j(z) \quad dz = \delta_{ij} D_j^z.$$