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Stochastic theory of zero-power nuclear reactors. Part 2. Probability of the branching process degeneration and issues of estimating the probability of a nuclear accident

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Abstract

A formula has been derived to estimate the degeneration probability in which the conditional probability that the fission process will not stop by the time t is determined by using a system of nonlinear differential equations presented in the paper. It follows from the formula that whatever the breeding ratio, a branching process with a probability of unity will never arise in a nuclear reactor without an external excitation.

A general relationship has been found for an asymptotic value of the probability that the fission process will stop in the nuclear reactor. It has been shown that Hansen's model is a quadratic approximation of the general model and always overestimates this probability.

The paper presents a theoretical analysis showing that Hansen's model underestimates the probability of a nuclear accident as compared to the estimates obtained by using the general model presented in the paper.

Keywords: Branching process; Degeneration probability; Neutron population; Uncontrolled reactor power excursion due to prompt neutrons; Nuclear accident; Model; Fission process; Self-sustaining fission chain reaction.

Probability for degeneration of branching process

Herein, the probability for degeneration of the branching process is understood as the probability that, by the time t, no particles of the T_1 and T_2 types (prompt neutrons and precursors of delayed neutrons) will remain in the reactor, with the number of the T_0-type particles (source neutrons) being random, if there was only one particle of the T_0 type in the reactor at the initial time $t = 0$ (all symbols used herein are the same as in part 1 of [1]). Such understanding of the degeneration probability somewhat differs from that being in common usage [2] and stems from the physical peculiarities of the branching process under investigation (we shall note that it is commonly considered in the theory of branching processes that the generation and death process tends to degenerate if particles of all types disappear).

The thing is that particles of the T_0 type are neutrons from an external source which are not generated in the neutron breeding process, that is, they do not appear from fission chains. Regardless of whether fission chains, i.e. particles of the T_1 and T_2 types, are present or not, the T_0-type particles are always present in the reactor in accordance with the distribution of their number depending on the external source power. If the fission chains (and, therefore, the T_1 and T_2 particles) disappear by the time t, regardless of if the T_0-type particles are present or not, it may be considered that the neutron generation and death branching process in the reactor, which started at the time $t = 0$ of the T_0 particle, had degenerated (stopped) by the time t.

Let the probability of this event be $P_0^{(0)}(t)$. It is clear that

$$P_0^{(0)}(t) = \sum_{\alpha_0} P_{\alpha_0,0,0}^{(0)}(t),$$

where $P_{\alpha_0,\alpha_1,\alpha_2}^{(0)}(t)$ is defined in [1]. The probabilities $P_{\alpha_0,0,0}^{(0)}(t)$ are defined from the generating function $P^{(0)}(t, s)$

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using formulas [2]

$$P^{(0)}_{0,0,0}(t) = \prod^{(0)}(t,0,0,0) = 0,$$

$$P^{(0)}_{1,0,0}(t) = \left.\frac{\partial \prod^{(0)}(t,s_0,0,0)}{\partial s_0}\right|_{s_0=0} = \left.\frac{1}{s_0}\prod^{(0)}(t,s_0,0,0)\right|_{s_0=0},$$

$$P^{(0)}_{\alpha_0,0,0}(t) = \left.\frac{\partial^{\alpha_0} \prod^{(0)}(t,s_0,0,0)}{\partial s^{\alpha_0}}\right|_{s_0=0} = 0 \quad (\alpha_0 > 1), \tag{1}$$

so

$$P^{(0)}_0(t) = \left.\frac{1}{s_0}\prod^{(0)}(t,s_0,0,0)\right|_{s_0=0}.$$

It can be seen from system of equations (5') in [1] that the first equation can be solved regardless of how the two others are solved. It has the form

$$\prod^{(0)}(t,s) = s_0 \exp\left(-S\int_0^t \left[1-\prod^{(1)}(\tau,s_1,s_2)\right]d\tau\right).$$

Here, the argument s_0 is omitted in record $\Pi^{(1)}$ for brevity, since $\Pi^{(1)}$ and $\Pi^{(2)}$ do not depend on s_0.

Therefore,

$$P^{(0)}_0(t) = \exp\left(-S\int_0^t \left[1-\prod^{(1)}(\tau,0,0)\right]d\tau\right). \tag{2}$$

The function $Q^{(1)}_B(t) = \prod^{(1)}(t,0,0)$ is the probability of the branching process degeneration by the time t, if it started at the time $t=0$ from one T_1-type particle. Accordingly, $Q^{(1)}_{\prod}(t) = 1 - Q^{(1)}_B(t)$ is the probability of the process continuation after the time t in the same condition.

Physically, the probabilities $Q^{(1)}_B(t)$ and $Q^{(1)}_{\prod}(t)$ are transparent enough. We shall create an "ideal" reactor fully isolated from external neutron sources (natural background, spontaneous fissions and so on). If a single neutron is somehow let into the reactor at the time $t=0$, the branching process starts because there is already one T_1-type particle in the reactor. The probabilities $Q^{(1)}_B(t)$ and $Q^{(1)}_{\prod}(t)$ are the probabilities that the branching process will come to an end or, accordingly, go on at the time $t>0$ in such "ideal" reactor.

One more initiation of the branching process in an "ideal" reactor may be assumed: one precursor of delayed neutrons (a T_2-type particle) is somehow let in the reactor at the time $t=0$. Then it is possible to consider the probabilities $Q^{(2)}_B(t)$, $Q^{(2)}_{\prod}(t) = 1 - Q^{(2)}_B(t)$ of the branching process degeneration and continuation respectively.

Thus, formula (2) can be rewritten in the form

$$P^{(0)}_0(t) = \exp[-S\gamma(t)], \tag{3}$$

where

$$\gamma(t) = \int_0^t Q^{(1)}_{\prod}(\tau)d\tau.$$

The probability $Q^{(1)}_{\prod}(t)$ is found in accordance with system of equations (5') in [1] from the system of nonlinear differential equations

$$\frac{dQ^{(1)}_{\prod}}{dt} = \frac{1}{L}\left\{\frac{k}{\bar{\nu}}\left(1-\sum_{\nu=0}^{N} r(\nu)\left[1-(1-\beta)Q^{(1)}_{\prod} - \beta Q^{(2)}_{\prod}\right]^{\nu}\right) - Q^{(1)}_{\prod}\right\}, \tag{4}$$

$$\frac{dQ^{(2)}_{\prod}}{dt} = \lambda\left(Q^{(1)}_{\prod} - Q^{(2)}_{\prod}\right)$$

with the initial conditions

$$Q^{(1)}_{\prod}(0) = Q^{(2)}_{\prod}(0) = 1. \tag{5}$$

According to formula (3), when $S=0$, that is, when there is no external neutron source in the reactor, $P_0{}^{(0)}(t) = 1$ for any $t \geq 0$. This means (which is fairly clear) that, *without being initiated externally, the branching process will never emerge with a probability of unity in the reactor with any breeding factor.*

As follows from [2], it can be shown that system of equations (4) remains valid in the event of a heterogeneous branching process, that is, when k is the function of the time t.

Probability of a nuclear accident

Hereinafter, nuclear accident is understood as an uncontrolled nuclear reactor power excursion due to prompt neutrons. Such nuclear accident takes place if at least one self-sustaining (time-infinite) fission chain occurs when the prompt breeding factor is equal to or greater than unity. Then the neutron population in the reactor grows unlimitedly in a very short time ($\sim L$) [3].

In terms of the nuclear accident probability evaluation, all fission chains initiated by source neutrons can be divided into two complementary classes: a class of non-self-sustaining fission (time-finite) chains and a class of self-sustaining fissions chains. The branching process degeneration probability $P^{(0)}_0(t)$, defined by formula (3), is also the probability that all fission chains in the reactor in the time interval [0, t] will turn out to be non-self-sustaining, or the probability that the time to the first self-sustaining fission chain (FSFC) will exceed t, so

$$\lim_{t\to\infty} P^{(0)}_0(t) = P^{(0)}_0(\infty)$$

there is a probability that no self-sustaining fission chains will occur in a finite time.

If $P^{(0)}_0(\infty) = 0$, one may talk about the distribution of the time to the FSFC. It is set by the distribution function $F(t) = P\{\tau \leq t\} = 1 - P^{(0)}_0(t)$. If $P^{(0)}_0(\infty) \neq 0$, one should talk about the conventional distribution of the given time, provided the FSFC occurred in a finite time. This conventional distribution is set by the relation

$$\left[1 - P^{(0)}_0(t)\right] / \left[1 - P^{(0)}_0(\infty)\right].$$