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Content knowledge and pedagogical content knowledge in Taiwanese and German mathematics teachers

Thilo Kleickmann [a,*], Dirk Richter [b], Mareike Kunter [c], Jürgen Elsner [d], Michael Besser [e], Stefan Krauss [f], Marie Cheo [g], Jürgen Baumert [d]

[a] Leibniz Institute for Science and Mathematics Education, Olshausenstraße 62, Kiel 24098, Germany
[b] Institute for Educational Progress (IQB), Hannoversche Straße 19, Berlin 10117, Germany
[c] Department of Psychology, University of Frankfurt a.M., Senckenberganlage 15, Frankfurt am Main 60325, Germany
[d] Max Planck Institute for Human Development, Lentzeallee 94, Berlin 14195, Germany
[e] University of Kassel, Department of Mathematics Education, Heinrich-Plett-Straße 40, Kassel 34132, Germany
[f] University of Regensburg, Department of Mathematics Education, Universitätsstraße 31, Regensburg 93053, Germany
[g] National Academy for Educational Research (NAER), No. 2, Sanshu Rd., Sanxia Township, Taipei County 23703, Taiwan

HIGHLIGHTS

- We assessed teachers' content knowledge and pedagogical content knowledge by tests.
- For these tests, previous studies showed predictive validity for student outcomes.
- The structure of subject matter knowledge was cross-culturally invariant.
- German and Taiwanese teachers' CK and PCK reflected differences in teacher education.
- CK and PCK in teacher subgroups reflected teacher selection processes.

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ABSTRACT

In comparing content knowledge (CK) and pedagogical content knowledge (PCK) of Taiwanese and German inservice mathematics teachers, the present study examines whether the two-dimensional structure of teachers' subject matter knowledge is cross-culturally invariant and whether differences in teacher education and in teacher selection are reflected in teachers' subject matter knowledge. The results confirm that CK and PCK represent two distinct, but correlated dimensions, even in teachers from completely different backgrounds. Taiwanese inservice teachers showed considerably higher CK and also higher PCK scores than German teachers. Teacher education and teacher selection should be considered important levers for reform in mathematics education.

1. Introduction

The notion that teachers need a deep understanding of the subject matter taught at schools as well as knowledge about how to make this content accessible to students was prominently made by Shulman (1987). Following Shulman's arguments, content knowledge and pedagogical content knowledge had been considered to be core components of teacher competence (Ball, Thames, & Phelps, 2008; Shulman, 1987). Recent studies showed that indeed content knowledge and pedagogical content knowledge impact instructional quality and student progress (Baumert et al., 2010; Hill, Rowan, & Ball, 2005). Thus, teachers' levels of CK and PCK represent key levers for educational reform.

Consequently, recent research addressed the question about the role of teacher education for teachers' subject matter knowledge, i.e. content knowledge (CK) and pedagogical content knowledge (PCK) (Kleickmann et al., 2013; Tatto et al., 2012). The international Teacher Education and Development Study (TEDS-M; Tatto et al., 2012), for instance, provided first evidence that structural

* Corresponding author. Leibniz Institute for Science and Mathematics Education, Department of Educational Research and Methodology, Olshausenstraße 62, Kiel 24118, Germany. Tel.: +49 431 880 3487.
E-mail address: kleickmann@ipn.uni-kiel.de (T. Kleickmann).

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differences in teacher education are related to preservice teachers' subject matter knowledge at the end of initial teacher education. In the present study, we extended this research to inservice (secondary) mathematics teachers. We focus on German and Taiwanese teachers because teacher education systems of both countries differ in two structural aspects: First, Taiwanese secondary teachers are prepared to teach only one subject, whereas their German colleagues are prepared to teach two subjects. The quantity of learning opportunities for PCK, and especially for CK, is larger in Taiwanese teacher education. Second, German teachers with a license to teach in upper secondary education are differently prepared than teachers with a license to teach only at middle schools. In Taiwan all secondary teachers (middle and upper secondary education) participate in the same teacher education programs. Teachers for upper secondary education are selected by a highly competitive process. We will go into these differences in more detail in Section 3. In the present study, we investigated whether these differences in teacher education and teacher selection are reflected in mathematics teachers' CK and PCK, and whether the structure of teachers' subject matter knowledge is invariant in teachers from these completely different backgrounds. From this binational comparison, we draw conclusions on the role of learning opportunities in the pre- and inservice phase for teachers' CK and PCK, as well as on the comparability of test scores in cross-cultural studies. In the following sections, we first define the concepts of CK and PCK, we then summarize literature on the structure of teachers' subject matter knowledge, and describe how it relates to student learning. Next, we provide an overview about teacher education and professional development in Germany and Taiwan, and explain how teachers are selected into training programs and different school types. At the end of the literature review, we summarize findings from TEDS-M (Tatto et al., 2012). This allows us to compare findings from TEDS-M at the end of initial teacher preparation with our findings on experienced mathematics teachers' CK and PCK.

2. Teacher knowledge of subject matter: definition, structure, and relations to student learning

CK and PCK represent aspects of teacher knowledge that are related to concrete topics taught at schools. Thus, the term subject matter knowledge is used to refer to both CK and PCK (Ball et al., 2008; Shulman, 1987). In the domain of mathematics, Ball and her colleagues introduced the term mathematical knowledge for teaching which also includes both CK and PCK (Ball et al., 2008). Although the definitions of CK and PCK vary across research groups (Hill et al., 2005; Krauss et al., 2008; Park & Oliver, 2008; Tatto et al., 2012), there seems to be consensus on some core aspects. Content knowledge (CK) represents teachers' understanding of the subject matter taught. According to Shulman (1986), "[t]he teacher need not only understand that something is so, the teacher must further understand why it is so" (p. 9). Thus, the emphasis is on a deep understanding of the subject matter taught. For instance, teachers need not only to know that $0.\overline{99} = 1$, but they have to be able to give reasons, why this is so (see Fig. A1). Pedagogical content knowledge (PCK) is the knowledge needed to make that subject matter accessible to students (Shulman, 1986, pp. 9–10). It includes knowledge of students' subject-specific conceptions and misconceptions as well as knowledge of subject-specific teaching strategies and explanations (see also Ball et al., 2008; Borko & Putnam, 1996; Depaepe, Verschaffel, & Kelchterman, 2013). For instance, if a student says: "I don't understand, why -1 times -1 equals 1" teachers need to know an explanation, that helps students to grasp the problem. If the teacher even knows multiple explanations, he or she will be even more likely to be able to support student understanding. The joint concept of mathematical knowledge for teaching (MKT) and the term subject matter knowledge that we use in the present paper highlight that CK and PCK are related to specific content. This implies that CK and PCK are related constructs, and that their empirical separability is not as clear as their theoretical distinction might suggest (Hill, Schilling, & Ball, 2004; Shulman, 1986). Factor analytical evidence suggests that CK and PCK represent two correlated, but separable and unique dimensions (Blömeke, Houang, & Suhl, 2011; Krauss et al., 2008; Phelps & Schilling, 2004). Some authors used nested factor models to disentangle common and specific variance in CK and PCK test items (Blömeke, Houang, et al., 2011; Brunner & Krauss, 2010; Hill et al., 2004). For instance, Hill et al. found items intended to measure CK and PCK to load on a common factor representing common content knowledge, but also on specific factors representing the unique variance of CK and PCK. However, in nested factor models on teachers' subject matter knowledge, the meaning of the specific PCK factor is somewhat unclear, since content knowledge is partialed out from that construct (Brunner & Krauss, 2010).

Confirmatory factor analysis (CFA) models also provided evidence for the multidimensionality of teachers' subject matter knowledge. In these models, CK and PCK represented separate, but medium to highly correlated factors (Hill et al., 2004; Krauss et al., 2008; Phelps & Schilling, 2004). Krauss et al. (2008) demonstrated that this correlation varied depending on the level of teacher knowledge. In particular, CK and PCK were strongly correlated ($r = 0.96$) in a group of teachers who received intensive training in mathematical content during initial teacher preparation. In this subgroup, CK and PCK were best represented by one single factor, i.e. they did not form separate knowledge dimensions. Hence the structure of teachers' subject matter knowledge is still an issue that needs further investigation.

Some evidence mainly from qualitative studies indicates that CK is a necessary condition for the development of PCK. For instance, a low level of individual CK constrains teachers in the interpretation of student thinking as well as in the construction of challenging learning situations (Capraro, Capraro, Parker, Kulm, & Raulerson, 2005; Haidar, 1997; Halim & Meraah, 2002; Van Driel, Verloop, & De Vos, 1998). However, it is assumed that CK is not sufficient for the development of PCK, and that CK needs to be transformed within deliberate learning situations (Friedrichsen et al., 2009; Kinach, 2002; Lee, Brown, Luft, & Roehrig, 2007).

Because teachers' core task is to make subject matter accessible to students, it is widely assumed that subject matter knowledge is at the heart of teacher competence (Ball, Lubienski, & Mewborn, 2001; Shulman, 1986; Woolfolk Hoy, Davis, & Pape, 2006). Indeed, recent studies provided strong, representative evidence that teachers' subject matter knowledge impacts both their instructional practice and their students' achievement gains in the domain of mathematics. Hill et al. (2005) found that elementary teachers' mathematical knowledge for teaching was substantially associated with student gains in mathematical understanding. Drawing on data from a longitudinal extension to the 2003 cycle of the OECD's Programme for International Student Assessment (PISA) in Germany, Baumert et al. (2010) showed that both PCK and CK affect student learning. However, despite the high correlation between CK and PCK, CK had lower predictive power for student progress than did PCK. Furthermore, PCK had the decisive impact on key aspects of instructional quality. Hence, teachers' subject matter knowledge represents a key target of teacher preparation and teacher selection.

3. Differences in German and Taiwanese teacher education and teacher selection processes

It is widely assumed that teacher education plays a key role in the development of teachers' CK and PCK (e.g., Borko & Putnam, 1996). Therefore, it is promising to investigate how the learning opportunities provided in different teacher education systems shape the construction of teachers' subject matter knowledge. For a