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Review

Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research

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HIGHLIGHTS

- PCK is differently conceptualized in empirical mathematics education research.
- Large-scale studies often measure PCK through a paper-and-pencil test.
- Small-scale studies typically use multiple qualitative data sources.
- Half of the PCK-studies investigates the development of (pre-service) teachers' PCK.

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ABSTRACT

Pedagogical content knowledge (PCK) was introduced by Shulman in 1986 and refers to the knowledge teachers use to translate particular subject matter to students, taking into account possible (mis)conceptions. PCK was – and still is – very influential in research on teaching and teacher education, mainly within the natural sciences. The present study aims at a systematic review of the way PCK was conceptualized and (empirically) studied in mathematics education research. Based on a systematic search in the databases Eric, PsycInfo and Web of Science 60 articles were reviewed. We identified different conceptualizations of PCK that in turn had a differential influence on the methods used in the study of PCK.

1. Introduction

The concept 'pedagogical content knowledge' (PCK) was introduced by Shulman (1986) as an answer to what he called a 'missing paradigm' in (research on) teaching and teacher education. Shulman criticized the lack of attention for subject matter both in the practice of training and evaluating pre-service teachers and in the research on effective teaching and teacher training practices. By introducing the concept PCK Shulman wanted to emphasize the central role of subject matter in (research on) teaching and teacher education and aimed at overcoming the artificial distinction between content and pedagogy.

Shulman (1987) identified PCK as one of the seven categories of teachers' knowledge base, defining it as "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8). The other six categories were: content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes, and values, and their philosophical and historical grounds. Of these seven categories, PCK was supposed to be "of special interest because it […] is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (p. 8). Shulman (1986) identified two components that are central to PCK, namely knowledge of instructional strategies and representations and knowledge of students' (mis)conceptions: "for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others […] [PCK] also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and

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backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates" (p. 9). Shulman's call for research on teachers' PCK closely connects with an older, European research tradition on 'subject matter didactics' (e.g., 'Fachdidaktik' in German, 'didactique spéciale' in French, 'vakdidactiek' in Dutch) (Van Driel & Berry, 2010). However, both research traditions have developed independently from each other, partly due to the negative connotation of 'didactics' in the Anglo-American educational research literature (Kansanen, 2009).[2]

Shulman's conceptualization of PCK – however – has also been criticized for a number of reasons. A first criticism relates to the lack of theoretical and empirical grounding for the existence of PCK as a distinct category in teachers' knowledge base (e.g., Ball, Thames, & Phelps, 2008; Bromme, 1995). In relation to this, Gess-Newsome (1999) made the distinction between the integrative model and the transformative model of teacher knowledge. Within the integrative model PCK does not exist as a separate phenomenon and teaching is considered to be the act of integrating knowledge across different knowledge domains, such as subject matter, pedagogy, and context. In contrast the transformative model, like in Shulman's conceptualization, treats PCK as a unique form of knowledge on which teachers rely while teaching. A second, and related criticism, is that Shulman held a static view on teachers' PCK. According to this view, PCK includes factual knowledge – knowledge about teaching – which can be acquired and applied independently from the classroom context. Other scholars took a more dynamic view on PCK, treating it essentially as a knowing-to-act that is inherently linked to and situated in the act of teaching within a particular context (Bednarz & Proulx, 2009; Hodgen, 2011; Mason, 2008; Petrou & Goulding, 2011). Third, scholars have doubted whether PCK can be theoretically and empirically distinguished from content knowledge (CK) (e.g., Baumert et al., 2010; Bednarz & Proulx, 2009; Blömeke, Felbrich, Müller, Kaiser, & Lehmann, 2008; Huillet, 2009; Marks, 1990; Saderholm, Ronau, & Brown, 2010). It is not surprising that mainly within a dynamic, integrated perspective on PCK that focuses on knowing-to-act at a particular moment in a particular context, the distinction between CK and PCK is criticized. It is claimed that one cannot meaningfully distinguish between CK and PCK since purely mathematical knowledge in the context of teaching simply does not exist (Huillet, 2009) and that the choices that teachers make in the act of teaching are always based on multiple dimensions, including a mathematical as well as a pedagogical aspect (Bednarz & Proulx, 2009). Fourth, some scholars have criticized Shulman's narrowing conceptualization of PCK in terms of teachers' knowledge of (1) instructional strategies and representations and (2) students' (mis)conceptions. They have argued for the need to broaden the concept in order to encompass, among others, curriculum knowledge (Grossman, 1990), beliefs (Friederichsen, Van Driel, & Abell, 2010), or emotions (Zembylas, 2007). Finally, scholars have argued that claims about what PCK as 'expert teaching' of particular subject matter should look like, are often normative (Ball et al., 2008; Bromme, 1995; Tirosh, Tsamir, Levenson, & Taback, 2011; Van Driel & Berry, 2012). What is accepted as PCK of experts is "in accordance with culturally accepted norms" (Tirosh et al., 2011, p. 129); international and/or national curriculum documents (such as the NCTM standards) shape how PCK is studied and interpreted.

Partly in an attempt to answer to these criticisms, several scholars have refined Shulman's conceptualization of PCK, both within Shulman's research group at Stanford University (Grossman, 1990; Marks, 1990) and at other research centers (e.g., Ball et al., 2008; Cochran, DeRuiter, & King, 1993; Hill, Ball, & Schilling, 2008; Hill, Schilling, & Ball, 2004). In their empirical research on the PCK of respectively language and mathematics teachers Grossman (1990) and Marks (1990) expanded the definition of PCK. According to Grossman (1990) four components are central to teachers' PCK: (1) knowledge of students' understanding, (2) knowledge of curriculum, (3) knowledge of instructional strategies, and (4) knowledge of purposes for teaching. Marks' study (1990) supported the following structure of PCK: (1) knowledge of students' understanding, (2) knowledge of media for instruction, (3) knowledge of subject matter, and (4) knowledge of instructional processes.

Other scholars have favored alternative conceptualizations. For instance, Cochran et al. (1993) used the term *pedagogical content knowing* (PCKg) instead of PCK to stress its dynamic nature. Moreover PCKg was conceptualized more broadly than PCK as it included "a teacher's integrated understanding of four components of pedagogy, subject matter content, student characteristics, and the environmental context of learning" (Cochran et al., 1993, p. 266).

Probably the most influential reconceptualization of teachers' PCK within mathematics education was done through the overarching constructs *mathematical knowledge for teaching* (MKT) or *content knowledge for teaching mathematics* (CKTM)[3] (e.g., Ball et al., 2008; Hill et al., 2004, 2008; Hill, Rowan, & Ball, 2005) that cover both CK and PCK. MKT refers to the mathematical knowledge that teachers need in order to teach mathematics. Yet, MKT differs from Shulman's conceptualization of PCK in at least two ways. First, Shulman's concept of PCK was initially purely theoretical, it served "as a heuristic, as a tool for helping the field to identify distinctions in teacher knowledge that could matter for effective teaching" (Ball et al., 2008, p. 392). In contrast, the concept MKT resulted from an attempt to refine and empirically validate PCK. Second, as reflected in Fig. 1, whereas PCK and CK are distinct categories in Shulman's (1986, 1987) conceptualization of teachers' knowledge base (see the left representation), PCK and CK are integrated within one overarching category of knowledge, MKT, that teachers need to teach mathematics (see the right representation). Moreover, curriculum knowledge, which is a separate category in Shulman's conceptualization of teacher's knowledge base is part of the PCK-components in Ball et al.'s (2008) MKT (i.e., knowledge of content and curriculum).

As shown in Fig. 1 MKT covers three categories that relate to teachers' CK: (1) common content knowledge (CCK, i.e., mathematical knowledge and skills used in settings other than teaching), (2) specialized content knowledge (SCK, i.e., mathematical knowledge and skills unique to teaching mathematics), and (3) horizon content knowledge (HCK, i.e., an awareness of how distinct mathematical topics are related to each other) (Ball et al., 2008). Another set of three categories within MKT concern teachers' PCK: (4) knowledge of content and students (KCS, i.e., knowledge about students' mathematical thinking, which requires an interaction between specific mathematical understanding and an understanding of students' mathematical thinking), (5) knowledge of content and

[2] In the Anglo-American research literature the concept 'didactic' is often associated with a traditional teacher-led approach and even moralizing form of lecturing. The Oxford Dictionary (10th Edition) explains the term 'didactic' as: "intended to teach, in particular having moral instruction as an ulterior motive. In the manner of a teacher; patronizing or hectoring."

[3] MKT and CKTM are used interchangeably (e.g., Hill et al., 2004). In the rest of this article we will use the term MKT.