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Teacher–student interaction in joint word problem solving. The role of situational and mathematical knowledge in mainstream classrooms

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HIGHLIGHTS

► 11 mainstream primary teachers were analyzed during word problem solving.
► Problems included additional relevant mathematical and situational information.
► Teachers did not use additional information to solve the problems in a meaningful way.
► Teachers could be preventing students from solving the problems in a meaningful way.

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ABSTRACT

Word problem solving involves the construction of two different mental representations, namely, mathematical and situational. Although educational research in word problem solving has documented different kinds of instruction at these levels, less is known about how both representational levels are evoked during word problem solving in day-to-day learning environments. The aim of this descriptive work is to analyze how mainstream teachers promote mathematical and situational knowledge while solving mathematically and situationally reworded word problems. The results suggest that word problem solving is faced by teachers as a mechanical and non-reflexive task which involves limited situational knowledge.

1. Introduction

Word problem solving is one of the mathematical school tasks most practiced around the world (Hiebert et al., 2003; Stigler & Hiebert, 1997). One reason for such frequency has to do with the significant role the task plays in developing meaning, which can be used for application of mathematical concepts and for integrating the real world in the mathematics classroom (Verschaffel, Greer, & De Corte, 2000). Another reason could be that word problem solving is considered a key component in learning mathematics (NCTM, 2000; OECD, 1999).

Furthermore, word problem solving has proved to be notoriously difficult for students, the so-called "black hole" for middle school mathematics (Bruer, 1994). This difficulty has been reported by international assessment programs. For example, *Trends in International Mathematics and Science Study* (TIMSS), *Program for International Student Assessment* (PISA), or *National Assessment of Educational Progress* (NAEP) have reported that students all over the world have difficulties when solving word problems, especially problems that are close to real-life situations (Hiebert et al., 2003; National Research Council, 1989; OECD, 2010; Stigler & Hiebert, 1997).

Although these reports are well known, significant changes have not taken place in the way in which word problems are usually approached in mainstream educational practice in schools (Cuban, 1993; Good, Clark, & Clark, 1997). It is therefore necessary to shed some light on what happens when students solve problems in the mainstream classroom in order to identify gaps between theoretical proposals and mainstream classroom practice. Specifically, the present study aims at analyzing how two specific aspects of problem solving, namely, mathematical and contextual knowledge, were evoked by 11 mainstream primary teachers when they solved

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two non-standard word problems jointly with their pupils in their mathematics class.

2. Theoretical framework

Word problem solving is a difficult task because to solve a word problem in a *genuine* way students must create different levels of mental representation (in the sense of Van Dijk & Kintsch, 1983). That is, solvers must create individual and subjective mental representations that relate to, firstly, events or situations (the so-called situational model); and secondly, the mathematical structure underlying the situation described (or mathematical model). In these mental models only the most relevant features of the problem, both situational (related to the qualitative situation described in the problem) and mathematical (related to numerical values and quantitative relations between them, in terms of the mathematical structure of the problem[2]) are represented (Verschaffel et al., 2000). During word problem solving students must generate an adequate situational model of the problem. They must consider the situation described by the problem and decide what information is essential and what information is less important. After the situational model has been created, the mathematical model of the problem must be generated by using the solver's previous mathematical knowledge to fit the situational model and the appropriate mathematical structure for the problem (in terms of quantities and the mathematical relations between them). Once this mathematical model has been constructed, the appropriate mathematical algorithms must be performed to obtain the result. Once the result is obtained, it must be interpreted in relation to the mathematical model and the real situation described in the problem. Finally, the interpreted and validated result needs to be communicated in a way consistent with the question of the problem.

However, students can also solve problems in a *superficial* way in which some steps of the *genuine* approach are by-passed. When using this *superficial* approach, there is no situational model of the problem, and the mathematical model is not based on mathematical reasoning. This is rather done in an automatic way by taking the data of the problem and selecting the algorithm to be employed using some meaningless strategy based on some salient element in the problem, like the *key word strategy* (e.g., Hegarty, Mayer, & Monk, 1995; Nesher & Teubal, 1975; Verschaffel, De Corte, & Pauwels, 1992). In this strategy, the algorithm to be employed is selected by using certain words of the problem as a hint (for example, "more" to add, "lose" to take away). Once the operation has been selected and the algorithm has been performed, the result is immediately communicated as the answer. Thus, students do not refer back to the original problem situation to verify that it is a meaningful response to the original question or to check its reasonableness. For example, considering the "bus problem" (Silver, Shapiro, & Deutsch, 1993), "*The Clearview Little League is going to a Pirates game. There are 540 people including players, coaches and parents. They will travel by bus, and each bus holds 40 people. How many buses will they need to get the game?*", the result of the division (540:40 = 13.5 buses) must be interpreted by considering the situation. Hence, the final answer must be, at least, 14 buses because a bus cannot be split into 0.5 buses. However, when using a superficial approach students give "13.5" as the answer without checking its reasonableness according to the situation described by the problem.

Previous research has shown that students usually face word problems in a *superficial* instead of a *genuine* way (Van Dooren, Verschaffel, Greer, & De Bock, 2006). Nevertheless, it is also well known that students can move from a *superficial* approach toward deeper approaches. For example, there is abundant evidence suggesting that instruction in the processes associated with word problem solving allows students to perform better. Thus we know that students improve their achievement when they operate with both the situational structure of the problem (Verschaffel & De Corte, 1997) and the mathematical structure of the problem, in terms of its change, compare or combine structure for addition and subtraction problems (see footnote[2]) (Fuson & Willis, 1989; Jitendra et al., 1998; Xin, Jitendra, & Deatline-Buchman, 2005). Furthermore, empirical studies that added extra information to the wording of the problem (the so-called "rewording") showed that the extra information might have a positive impact on students' problem-solving processes and skills, especially when problems are mathematically difficult.[3] Mathematically reworded problems aim at clarifying the mathematical relations between the sets implied in the problem (Cummins, 1991; Davis-Dorsey, Ross, & Morrison, 1991; De Corte & Verschaffel, 1985; Vicente, Orrantia, & Verschaffel, 2007, 2008a,b) and situationally reworded problems present the (real world) situation referred to by the text of the problem in a more enriched and elaborate way (Coquin-Viennot & Moreau, 2007; Cummins, Kintsch, Reusser, & Weimer, 1988; Orrantia, Tarín, & Vicente, 2011; Staub & Reusser, 1992; Stern & Lehrndorfer, 1992).

Despite the fact that reworded problems might help students to perform better, few studies have analyzed how situational and mathematical models construction is promoted during word problem solving in day-by-day classroom practices. Chapman (2006) probably provides the best description of the role of promoting mathematical and situational models construction in word problem solving. She explored how social/cultural contexts are used in instruction in mathematics classrooms, and based on the modes of knowing proposed by Bruner (1985, 1986), she considered two different modes of going about the task: a paradigmatic mode and a narrative mode. When the paradigmatic mode is used, teachers focus on the mathematical aspects of the problem, such as data selection or mathematical reasoning, which are relatively context-free. That is, this approach focuses students' attention on strategies and ways of thinking that are independent of a particular real-life context. In contrast, when the narrative mode is used, teachers focus on the surrounding context of the problem, that is, they address the cover story of the word problem in order to understand or relate the storyline, plot, characters, objects, situations, actions, relationships or intentions to attain

[2] Following Riley and Greeno (1988), for addition and subtraction problems liked those used in this study, three mathematical structures can be distinguished: change, compare and combine. Change problems are those in which an initial quantity is increased or decreased by a change quantity to result in a final quantity (i.e., "John had 5 marbles. He won 2 marbles in a game. How many marbles does John have now?"). In compare problems a compare quantity is compared do a reference quantity, there being a quantitative difference between them (i.e.: "John has 5 marbles. Peter has 3 marbles more than John. How many marbles does Peter have?"). Finally, in combine problems there are two quantities or parts that are combined into a whole quantity (i.e.: "John has 5 marbles. Peter has 3 marbles. How many marbles do John and Peter have altogether?").

[3] Following Riley and Greeno (1988), for addition and subtraction problems the most difficult ones are: a) change problems with the initial quantity unknown ("John had some marbles. He won 2 marbles in a game. Now he has 8 marbles. How many marbles did John have at the beginning"); b) compare problem with the reference quantity unknown ("John has 5 marbles. John has 3 marbles more than Peter. How many marbles does Peter have?"); and c), combine problems with the whole quantity unknown ("John and Peter have 5 marbles altogether. Peter has 3 marbles. How many marbles does John have?" The reader can compare the difficulty of all these examples to those in the previous footnote, which were the easiest of each problem type.