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Construction of system of spheres-based transitively relational partial meet multiple contractions: An impossibility result

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ABSTRACT

In this paper we show that, contrary to what is the case in what concerns contractions by a single sentence, there is not a system of spheres-based construction of multiple contractions which generates each and every transitively relational partial meet multiple contraction.

Before proving the general result, we consider the class of system of spheres-based multiple contractions introduced in [17,5] and show that this class neither subsumes nor is subsumed by the class of transitively relational partial meet multiple contractions.

Furthermore, we propose two system of spheres-based constructions of multiple contractions which generate (only) transitively relational partial meet multiple contractions. Therefore we can conclude that, although it is impossible to obtain a system of spheres-based definition of all the transitively relational partial meet multiple contractions, there are classes of system of spheres-based multiple contractions which are subsumed by the class of transitively relational partial meet multiple contractions.

1. Introduction

In the belief revision literature the *partial meet contraction*, introduced in the seminal paper [1], constitutes the standard model of belief contraction functions. The main purpose of such framework — which is commonly known as AGM contraction — is modelling the dynamics of the set of beliefs of an agent. More precisely, the AGM model essentially provides a definition for a class of *contraction functions* that receive a belief set — a logically closed set of sentences — and a sentence, and return a belief set which is a subset of the original one that does not contain the received sentence. A possible worlds semantics for partial meet contractions (i.e. a characterization of those functions in terms of possible worlds) was proposed in [8,12]. Furthermore, based on such semantics, Grove [8] presented a way of defining contraction function by means of a system of spheres — the so-called *system of spheres-based contractions*. In that same paper it was shown that such class of functions coincides with the class of *transitively relational partial meet contractions*, a special (proper) subclass of *partial meet contractions* which was also introduced in [1].

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A natural generalization of the above mentioned contraction functions is to allow the epistemic input to be *a set of sentences* rather than a *single* sentence. In this case, the new belief set is a subset of the original belief set that does not contain the set of sentences given as input. In [7], Fuhrmann and Hansson remarked that two kinds of such functions may be considered, namely, on the one hand, *package contractions* which are operations that account for the removal of all the sentences of a given set from the original belief set and, on the other hand, *choice contractions* which are functions that, receiving as input a belief set and a set of sentences, return as output a belief set which is contained in the input belief set and does not contain at least one of the sentences of the received set. In that same paper, it was shown that when the set to be contracted is finite, the choice contraction can be reduced to a contraction by a single sentence (namely by the conjunction of all the elements from that set). Since in this paper we study only contractions by finite sets of sentences, we will only consider package contractions, which from now on we shall call *multiple contractions*. We shall often use the expression *singleton contraction* to refer to an operation of contraction by a single sentence.

Multiple contraction is an important type of belief change that has been studied extensively in the literature since the early '90s [14,7,23,16,18,5,6,19]. Nowadays its significance has increased even further, partially due to the emergence of intelligent agents (softbots, robots, etc.) which typically receive input from more than one source simultaneously (for example, through several sensors). We note that such scenarios are outside the scope of classical belief change operators that can only handle changes by a single input at a time.

The generalization of partial meet singleton contractions was originally presented in [9,10] where the class of *partial meet multiple contractions* was introduced. Afterwards, in [17,18] the possible worlds semantics for such functions was provided (which can be seen as a generalization to the multiple contraction level of the possible worlds semantics for partial meet singleton contraction). In [17,5] the class of *spheres filtration-based multiple contractions* was presented,[1] which is a generalization to the multiple contraction case of Grove's class of system of spheres-based singleton contractions. Later, in [6,19] three different axiomatic characterizations for this class of multiple contractions were presented.

Furthermore, in [17,5] it was shown that every spheres filtration-based multiple contraction is a partial meet multiple contraction. However, in order to verify whether there is a complete analogy between the proposed class of spheres filtration-based multiple contractions and Grove's class of system of spheres-based (singleton) contractions it remains to analyse if the class of spheres filtration-based multiple contractions coincides with the class of transitively relational partial meet multiple contractions (as it is the case regarding their singleton contraction counterparts).

In this paper we will show that this does not hold and, more generally, we will prove that it is in fact impossible to obtain a system of spheres-based definition of multiple contraction functions which encompasses all the transitively relational partial meet multiple contractions.

More precisely, we will start by showing that the class of spheres filtration-based multiple contractions (proposed in [17,5]) neither contains nor is contained in the class of transitively relational partial meet multiple contractions. This conclusion naturally leads to question if there is a (different) system of spheres-based definition of multiple contractions which generates precisely the class of transitively relational partial meet multiple contractions or at least a class of functions which is contained in it. We shall address this issue as follows. First we will prove that it is not possible to construct all the transitively relational partial meet multiple contractions by means of a system of spheres-based method. Afterwards we will present two system of spheres-based methods for constructing multiple contractions and show that both those methods give rise (only) to transitively relational partial meet multiple contractions.

The paper is organized as follows: In Section 2 we recall all the background needed for the rest of the paper. In Section 3 we show, by means of two counterexamples that neither all spheres filtration-based multiple contractions are transitively relational partial meet multiple contractions nor the converse holds. Afterwards, in Section 4 we show that, in general, using a system of spheres-based approach it is not possible to obtain all the transitively relational partial meet multiple contractions. Then, in Section 5 we propose two system of spheres-based definitions of partial meet multiple contraction functions, which give rise (only) to transitively relational partial meet multiple contractions. Subsequently, in Section 6 we briefly describe some works that can be found in the literature and whose topics are closely related to the one of the present paper. Finally, in Section 7 we summarize the main contributions of the paper and identify some open questions related to the topic of this paper.

1.1. Basic notations and conventions

We will assume a language $\mathcal{L}$ that is built from a finite set of propositional symbols and the Boolean connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$. We shall make use of a consequence operation *Cn* that takes sets of sentences to sets of sentences and which satisfies the standard Tarskian properties, namely *inclusion, monotony* and *iteration*. Furthermore we will assume that *Cn* satisfies *supraclassicality, compactness* and *deduction*. We will sometimes use $Cn(\alpha)$ for $Cn(\{\alpha\})$, $A \vdash \alpha$ for $\alpha \in Cn(A)$, $\vdash \alpha$ for $\alpha \in Cn(\emptyset)$, $A \nvdash \alpha$ for $\alpha \notin Cn(A)$, $\nvdash \alpha$ for $\alpha \notin Cn(\emptyset)$. The letters $\alpha, \alpha_i, \beta, \dots$ (except for γ) will be used to denote sentences. $\top$ stands for an arbitrary tautology and $\bot$ for an arbitrary contradiction. $A, A_i, B, \dots$ shall denote sets of sentences of $\mathcal{L}$. $\mathbf{K}$ is reserved to represent a set of sentences that is closed under logical consequence (i.e. $\mathbf{K} = Cn(\mathbf{K})$) — such a set is called

[1] In [17,5] these functions were designated by *system of spheres-based multiple contractions*, however here it is convenient to use this less general denomination.