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Semantic characterization of rational closure: From propositional logic to description logics

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ABSTRACT

In this paper we provide a semantic reconstruction of rational closure. We first consider rational closure as defined by Lehman and Magidor [33] for propositional logic, and we provide a semantic characterization based on a minimal models mechanism on rational models. Then we extend the whole formalism and semantics to Description Logics, by focusing our attention to the standard $\mathcal{ALC}$: we first naturally adapt to Description Logics Lehman and Magidor's propositional rational closure, starting from an extension of $\mathcal{ALC}$ with a typicality operator **T** that selects the most typical instances of a concept C (hence $\mathbf{T}(C)$ stands for typical C). Then, for the Description Logics, we define a minimal model semantics for the logic $\mathcal{ALC}$ and we show that it provides a semantic characterization for the rational closure of a Knowledge base. We consider both the rational closure of the TBox and the rational closure of the ABox.

1. Introduction

In [31] Kraus Lehmann and Magidor (henceforth KLM) proposed an axiomatic approach to nonmonotonic reasoning based on the notion of plausible inference. Plausible inferences are represented by conditionals of the form $A \mid\sim B$, to be read as "typically or normally A entails B". For instance, the conditional assertion $\mathit{monday} \mid\sim \mathit{go\_work}$ can be used in order to represent that "normally if it is Monday I go to work". Conditional entailment is nonmonotonic since from $A \mid\sim B$ one cannot derive $A \wedge C \mid\sim B$, in our example from $\mathit{monday} \mid\sim \mathit{go\_work}$ one cannot monotonically derive $\mathit{monday} \wedge \mathit{ill} \mid\sim \mathit{go\_work}$ ("normally if it is Monday, even if I am ill I go to work").

KLM presented a hierarchy of axiomatic systems for plausible inference, each system specifies a set of postulates characterizing plausible inference. The systems are, from the weakest to the strongest: cumulative logic C, loop-cumulative logic CL, and most important preferential logic P. In subsequent work [33] preferential logic was strengthened to rational logic R and the latter was proposed as the most adequate system to represent (nonmonotonic) plausible inference.

Although it is arguable whether, KLM systems, and in particular R, represent adequately all types of nonmonotonic inferences,[1] we think that KLM systems and the strongest R in particular, are still a significant proposal for nonmonotonic

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[1] It has been shown that existing nonmonotonic systems do not satisfy in general all the properties of KLM systems: in particular circumscription (for well-founded theories) satisfies all postulates of preferential logic, but it does not satisfy rational monotony of R, whereas default logic fails to satisfy even

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reasoning for two reasons: (a) on a theoretical level, they define a set of inferential properties which are useful (even if not necessarily wanted) to classify and analyze concrete nonmonotonic inference, (b) they provide a simple and *direct* language to express plausible inferences and to reason about them.

In this work we take KLM logic R as the basis of our approach to nonmonotonic reasoning. Even if R formalizes some properties of nonmonotonic inference it is too weak in itself to perform useful nonmonotonic inferences.

We have just seen that by the nonmonotonicity of $\mid\!\sim$, $A \mid\!\sim B$ does not entail $A \wedge C \mid\!\sim B$ ($monday \mid\!\sim go\_work$ does not entail $monday \wedge ill \mid\!\sim go\_work$), and this is a wanted property of $\mid\!\sim$: it is what allows to express sets of conditionals that in classical logic would lead to contradictory or absurd conclusions (for instance $\{monday \rightarrow go\_work, monday \wedge ill \rightarrow \neg go\_work\}$ gives $\neg(monday \wedge ill)$ in classical logic, that is that it is impossible to be ill on Monday). However, there are cases in which, in the absence of information to the contrary, we would like to be able to tentatively infer that also $A \wedge C \mid\!\sim B$, with the possibility of withdrawing the inference in case we discovered that it is inconsistent. For instance, we might want to infer that $A \wedge C \mid\!\sim B$ when C is irrelevant with respect to the property B: in the example, we might want to tentatively infer from $monday \mid\!\sim go\_work$ ("normally if it is Monday, I go to work") that $monday \wedge shines \mid\!\sim go\_work$ ("normally if it is Monday, even if the sun shines I go to work"), with the possibility of withdrawing the conclusion if we discovered that indeed the sun shining prevents from going to work. R cannot handle irrelevant information in conditionals, and the inferences just exemplified are not supported.

Partially motivated by this weakness, Lehmann and Magidor have proposed a true nonmonotonic mechanism on the top of R. *Rational closure* [33] on the one hand preserves the properties of R, on the other hand it allows to perform some truthful nonmonotonic inferences, like the one just mentioned ($monday \wedge shines \mid\!\sim go\_work$). In [33] the authors give a syntactic procedure to calculate the set of conditionals entailed by the rational closure as well as a quite complex semantic construction. It is worth noticing that a strongly related construction has been proposed by Pearl [38] with his notion of 1-entailment, originating from a probabilistic interpretation of conditionals within the well-established System Z.

In this paper we provide a semantic reconstruction of rational closure for propositional logic as well as for Description Logics (DLs for short) with a specific attention to the standard $\mathcal{ALC}$. We first consider rational closure as defined by Lehman and Magidor [33] for propositional logic, and we provide a semantic characterization based on a minimal models mechanism on rational models. Then we extend the whole formalism and semantics to Description Logics: we first naturally adapt to DLs Lehman and Magidor's propositional rational closure, starting from an extension of $\mathcal{ALC}$ with a typicality operator $\mathbf{T}$ that selects the most typical instances of a concept C (the extension is called $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$). For $\mathcal{ALC} + \mathbf{T}_{\mathsf{R}}$, we provide both a syntactic and a semantical notion of rational closure, along the same lines used for the propositional case: we first define rational closure over the TBox, and subsequently rational closure for the ABox.

The first problem we tackle in this work is that of giving a purely semantical characterization of the syntactic notion of rational closure. Our semantic characterization has as its main ingredient the modal semantics of logic R, over which we build a minimal models' mechanism, based on the minimization of the rank of worlds. Intuitively, we prefer the models that minimize the rank of domain elements: the lower the rank of a world, the more normal (or less exceptional) is the world and our minimization corresponds intuitively to the idea of minimizing less-normal or less-plausible worlds (or maximizing most plausible ones). We show that a semantic reconstruction of rational closure can be obtained as a specific instance of a general semantic framework for nonmonotonic reasoning. Within this general framework we give two characterizations of rational closure: one based on a fixed interpretations semantics and the other with a variable interpretations semantics.

The theoretical question we address in this first part of the paper is the following:

A) Given the fact that logic R is characterized by a specific class of Kripke models, what are the Kripke models that characterize the rational closure of a set of positive conditionals?

We notice in passim that our semantic characterization of rational closure in terms of minimal models is different from the one given by Lehmann and Magidor's in [33] which is based on a different notion of minimal models. Moreover we consider our semantic characterization as a specific case of a general minimal models' mechanism for nonmonotonic reasoning, and in this paper we show under what conditions we capture rational closure. The generality of our semantical characterization is well-suited to study variants of rational closure. Finally, the semantic characterization does also easily extend to other logics, as Description Logics ($\mathcal{ALC}$), that we discuss next. In the second part of the paper we consider Description Logics. If propositional KLM systems deal with propositions ("I go to work") and relations among propositions ("usually, if it is Monday, then I go to work"), Description Logics deal with concepts, relations among concepts, as well as with individuals. In Description Logics one can use concept inclusion in order to express that all the members of a class have a given property (thus $Cats \sqsubseteq Mammal$ expresses the general property that "cats are mammals", and $Pet \sqsubseteq \exists HasOwner.\top$ that "all pets have an owner"). One can also use assertions in order to represent the fact that an individual has a given property, e.g. $Cat(tom)$ ("Tom is a cat") or $\exists HasOwner.\top(tom)$ ("Tom has an owner") or $HasOwner(tom, nadeem)$ ("Nadeem is Tom's owner"). A distinguishing quality of Description Logics is their controlled complexity: the trade-off be-

the cumulativity postulate of the weakest logic C. Of course, a nonmonotonic mechanism may give rise to different inference relations (skeptical, credulous, etc.) with different properties.