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# Measuring inconsistency in probabilistic logic: rationality postulates and Dutch book interpretation

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ABSTRACT

Inconsistency measures have been proposed as a way to manage inconsistent knowledge bases in the AI community. To deal with inconsistencies in the context of conditional probabilistic logics, rationality postulates and computational efficiency have driven the formulation of inconsistency measures. Independently, investigations in formal epistemology have used the betting concept of Dutch book to measure an agent's degree of incoherence. In this paper, we show the impossibility of joint satisfiability of the proposed postulates, proposing to replace them by more suitable ones. Thus we reconcile the rationality postulates for inconsistency measures in probabilistic bases and show that several inconsistency measures suggested in the literature and computable with linear programs satisfy the reconciled postulates. Additionally, we give an interpretation for these feasible measures based on the formal epistemology concept of Dutch book, bridging the views of two so far separate communities in AI and Philosophy. In particular, we show that incoherence degrees in formal epistemology may lead to novel approaches to inconsistency measures in the AI view.

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## 1. Introduction

*"when you can measure what you are speaking about, you know something about it; but when you cannot [...] your knowledge is of a meagre and unsatisfactory kind;"*

— Lord Kelvin [45]

Measuring has been a prominent activity in advancing scientific and technological development. Not all measures are alike and good measures express intuitive notions in a useful way. In the field of deductive logical reasoning, one usually has an intuition expressing that one theory is *more inconsistent* than other, capturing the idea that the "effort" to restore consistency is greater in one case than the other. Also, no effort is required to restore the consistency of a consistent theory.

Based on those intuitions, there are several proposals for measuring inconsistency in knowledge bases over purely logical languages [19]. Some of these proposals involved attaching probabilities to formulas [29], or the combination of inconsistency factors [20]. Some of these measures are discrete or even qualitative, while others are more like distances, but all these measures have to behave like an *information measure* [6]. And to adhere to certain intuitions, a series of postulates

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for inconsistency measures for purely logical knowledge bases were proposed [21,22]; for example, the *consistency postulate* states that the inconsistency measure of a consistent base is 0.

Purely logical bases are known to be expressively limited in representing uncertainty required for real-world applications. In this work, we are interested in measuring the inconsistency of knowledge bases over logical probabilistic languages, which combine the deductive power of logical systems with the well-founded theory of probability. This kind of extension of purely logical systems can be traced back to the work of Boole [2], but has gained attention of AI researchers since the work of Nilsson [33], and has been extended to conditional probabilistic logic [37].

In AI, one of the main uses of measuring inconsistency in a knowledge base is to guide the consolidation of inconsistent pieces of information. Within propositional logic, Grant and Hunter [13] showed how inconsistency measures can be used to direct the stepwise resolution of conflicts via the weakening or the discarding of formulas.

In probabilistic bases, inconsistencies are rather common, specially when knowledge is gathered from different sources. To fix these probabilistic knowledge bases, one can, for instance, delete pieces of information, or change the probabilities' numeric values (or intervals). In this case, an inconsistency measure helps one to detect if a change approximates consistency or not. In other areas, inconsistency measures for probabilistic logic have found applications in merging conflicting opinions, leading to an increased predictive power [47,25], and in quantifying the incoherence of procedures from classical statistical hypothesis testing [41].

**Example 1.1.** Consider we are devising an expert system to assist medical diagnosis. Suppose a group of experts on a disease $D$ is required to quantify the relationship between $D$ and its symptoms. Suppose three conditional probabilities are presented:

- the probability of a patient exhibiting symptom $S_1$ given he/she has disease $D$ is 50%;
- the probability of a patient exhibiting symptom $S_2$ given he/she exhibits symptom $S_1$ and has disease $D$ is 80%;
- the probability of a patient exhibiting symptom $S_2$ given he/she has disease $D$ is 30%.

A knowledge engineer, while checking those facts, finds that they are inconsistent: according to the first two items, the probability of symptom $S_2$, given disease $D$, should be at least $50\% \times 80\% = 40\%$, instead of 30%. He does not even know where each probability came from, but plans to change the probabilities, since consistency is a requirement. How should he proceed? Which probabilities is the degree of inconsistency most sensitive to? Once chosen which number to change, should it be raised or lowered in order to approximate consistency? These are the kind of questions an inconsistency measure can help to answer.

The issue of measuring inconsistency in probabilistic bases has more recently been tackled by Thimm [44], Muiño [31] and Potyka [34], who developed measures based on distance minimization, tailored to the probabilistic case. Potyka focused on computational aspects, looking for efficiently computable measures [34]. Muiño was driven by the CADIAG-2 knowledge base, presenting its infinitesimal inconsistency degree, however based on a different semantics [31]. Thimm [44] adapted Hunter and Konieczny's [22] desirable properties for inconsistency measures to the probabilistic setting, developing measures that satisfy a set of rationality postulates.

It was Thimm [44] who realized the importance of *continuity* as a Postulate for the probabilistic case, namely the property that a small change in the probability associated to formula (absent in the purely logical case) should lead only to small changes in the inconsistency measure. It was just natural that, (conditional) probabilistic logic being an extension of the classical cases, the continuity postulate was simply added to the postulates defining classical inconsistency measures.

In this work, we argue that continuity cannot hold together with classical postulates such as consistency and independence, and some of these postulates must be abandoned or exchanged for other ones that restore joint satisfiability. So the first contribution of this work is that we *identify* and *fix* the possible problem with the postulates proposed by Thimm [44].

Another contribution lies in showing that these measures of inconsistency have a direct counterpart in formal epistemology research over the coherence of an agent's degrees of belief. It is known that inconsistent probabilistic beliefs correspond to a set of bets with guaranteed loss to the agent, which is called a "Dutch Book" [8,27]. This agent's incoherence has been measured by formalizing the intuition that the greater the inconsistency the greater the corresponding sure loss, and vice versa [40,43]. Thus we interpret these incoherence measures via guaranteed losses as inconsistency measures, showing that existing measures based on distance minimization correspond to guaranteed losses that quantify an agent's incoherence. To the best of our knowledge, no clear link has been shown between these two areas.

Here is a bird's-eye view of how we achieve these goals.

After introducing probabilistic knowledge bases in Section 2, this paper develops three main contributions, in three different sections, dealing closely with three other works. In the following, we overview such contributions, together with the organization of the paper and their relation to the existing literature.

Inconsistency measures for probabilistic knowledge bases were analyzed via rationality postulates by Thimm [44]. In Section 3, we argue for the incompatibility of such desirable properties. Firstly, we introduce the problematic postulates: consistency, independence and continuity. The *independence postulate* claims that a free conditional — a (conditional) probability assignment that does not belong to any minimal inconsistent set — can be rule out without changing the degree of

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