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Differential evolution for noisy multiobjective optimization

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ABSTRACT

We propose an extension of multiobjective optimization realized with the differential evolution algorithm to handle the effect of noise in objective functions. The proposed extension offers three merits with respect to its traditional counterpart. First, an adaptive selection of the sample size for the periodic fitness evaluation of a trial solution based on the fitness variance in its local neighborhood is proposed. This avoids the computational complexity associated with the unnecessary reevaluation of quality solutions without disregarding the necessary evaluations for relatively poor solutions to ensure accuracy in fitness estimates. The second strategy is concerned with determining the expected value of the noisy fitness samples on the basis of their distribution, instead of their conventional averaging, as the fitness measure of the trial solutions. Finally, a new crowding-distance-induced probabilistic selection criterion is devised to promote quality solutions from the same rank candidate pool to the next generation, ensuring the population quality and diversity in the objective spaces. Computer simulations performed on a noisy version of a well-known set of 23 benchmark functions reveal that the proposed algorithm outperforms its competitors with respect to inverted generational distance, spacing, error ratio, and hypervolume ratio metrics.

1. Introduction

The multiobjective optimization (MOO) literature has witnessed a radically different perspective in solving real-world problems using evolutionary computing methods. A MOO is concerned with mathematical optimization problems involving two or more complex, nonlinear, conflicting objectives to be optimized simultaneously. Usually, a derivative-free single-objective optimization algorithm generates new trial solutions that are biased toward the better region of the objective space, and weeds out poor solutions using a competitive selection over iterations. However, for a nontrivial MOO problem, there exists no single solution that simultaneously optimizes each objective. To jointly optimize multiple objective functions in a MOO [1], selection of trial solutions is performed by Pareto ranking, which is concerned with judiciously identifying nondominated trial solutions from the rest of the population. Pareto ranking is induced by the fitness measure of all objective functions for individual trial solutions.

The objectives, being functions of certain variables describing a specific problem, usually return a unique value for the variables in their argument. However, in many scientific/engineering problems, it has been observed that even though the measurements of the variables remain constant, the objective functions return different values because of noise-induced dynamic variation of the objective surfaces. This class of problem is referred to as the “noisy optimization problem.” Noise

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creeps into the picture because of technological limitations, modeling errors, and incomplete data, leading to different results from repeated evaluations for the same set of parameter values of the objective functions. In such circumstances, a quality trial solution in a MOO may be deprived of being promoted to the next generation because of its poor (noisy) fitness estimates, while a deceptive solution with illusive good fitness may not be discarded from the current population [2,3].

This paper addresses the issues of uncertainty management (regarding the selection of qualitative trial solutions) in MOO in the presence of noise by incorporating the following three policies: adaptation of the sample size of a trial solution for its periodic fitness evaluation, expected fitness estimation from the measured noisy fitness samples, and crowding-distance-induced stochastic selection. First, the sample size for periodic fitness evaluation of each trial solution is adapted by means of the fitness variance in their local neighborhood. "Sampling" refers to the periodic fitness evaluation of a trial solution to diminish the risk of promoting inferior solutions in the noisy environment. It is worth mentioning that the adaptive selection of the sample size is momentous as increasing the sample size augments the quality measure of fitness at the cost of additional runtime. Here a nonlinear form (capturing the relationship between the sample size of a trial solution and the fitness variance in its local neighborhood) induced by an exponential function is regarded to efficiently balance the trade-off between runtime complexity and computational accuracy.

Second, while measuring the fitness of a trial solution, traditional methods [4–6] refer to the average fitness of the samples. However, the average fitness presumes equal probability of occurrence of all fitness samples, and thus returns a poor fitness estimate when the noise variance (in the fitness measure of the solutions) in the local neighborhood of a selected trial solution is large. This problem is circumvented here by referring to the expected value of the fitness samples as the true fitness estimate of a trial solution. The expected fitness concerned with the occurrence probability of the fitness samples seems to give a better fitness measure of a given trial solution. We introduce a novel strategy to evaluate the expected fitness of the trial solutions from the distribution of the fitness samples in the entire sample space. In the present context, a density-based nonuniform partitioning of the fitness sample space is employed to capture the uncertainty involved in the fitness measurement of the noisy fitness samples.

Finally, we develop a probabilistic selection (PS) policy to encapsulate the diversity as well as the quality of the non-dominated trial solutions even in noisy fitness landscapes. It is observed that the deterministic selection scheme of the crowding-distance-based sorting (used for promoting trial solutions from the same rank candidate pool to the next generation), which is employed in traditional MOO algorithms, can lead to suboptimal or misleading sets of nondominated solutions in the noisy environment even when sampling is used [7]. The selection strategy here depends not only on the density of nondominated solutions surrounding an individual in the objective space, but also on the reliability of its measured fitness samples. We develop a new probabilistic measure of the reliability based on the skewness of the distribution of the fitness samples. The degree of asymmetry of the distribution of the fitness samples is captured by skewness. Consequently, it provides a unique approach for identifying the rare fitness samples lying in the tail of the distribution. These infrequent samples (far away from the expected fitness) are assumed to occur because of the creeping of noise in the fitness landscapes. The rarer the occurrence of the infrequent samples (i.e., the closer the fitness samples are to the expected value with small skewness of the distribution), the greater is the degree of credibility of the fitness estimates of a given trial solution. The trial solutions having a greater crowding distance and a high grade of reliability (assessed using the probability of occurrence of no rare samples) are given more precedence during ranking of solutions in the same front.

The evolutionary component of the proposed noisy MOO algorithm has been realized here by the differential evolution for MOO (DEMO) [8] algorithm for its proven merits in global optimization. Some of the attractive features of DEMO justifying its selection in the design of the proposed noisy optimization algorithm include the simplicity of its structure leading to ease of coding, very few control parameters, and faster convergence [48,49] in comparison with other MOO algorithms.

Performance analysis of the proposed noisy optimization algorithm realized with DEMO—referred to as "differential evolution for noisy MOO" (DENMO) henceforth—is studied using the noisy version of a set of 23 benchmark functions. Experiments were undertaken to compare the potency of the proposed algorithm with differential evolution for MOO with noise (DEMON) [9], nondominated sorting genetic algorithm II (NSGA-II) with α-dominance operator (NSGA-II-A) [10], confidence-based dynamic resampling (CDR) [11], simulated annealing for noisy MOO [12], elitist evolutionary multiagent system [13], multiobjective evolutionary algorithm with robust features (MOEA-RF) [14], modified NSGA-II [7], noise-tolerant strength Pareto evolutionary algorithm [15], and Pareto front-efficient global optimization [16]. In this study, the objective functions are contaminated with noise samples taken from five noise distributions—namely, Gaussian, Poisson, Rayleigh, exponential, and random (with positive and negative expeditions of the noise amplitude within $\pm 25\%$ of the true fitness function values). Experiments reveal that the proposed realization outperforms other algorithms for four important performance metrics—that is, *inverted generational distance* (*IGD*), *spacing*, *error ratio* (*ER*), and *hypervolume ratio* (*HVR*).

The paper is divided into seven sections. Section 2 briefly reviews the literature on the strategies adopted by evolutionary algorithms to solve noisy MOO problems. In Section 3, we provide an overview of MOO and the DEMO algorithm. Section 4 provides the noise handling mechanism in DENMO. The experimental settings for the benchmarks and the simulation strategies are explained in Sections 5 and 6, respectively. Conclusions are given in Section 7.

2. Literature review

Recently, researchers have been very interested in developing robust MOO algorithms that can search for optimal solutions even when deceived by noise. Stagge et al. [17] employed the concept of *sampling* (fitness reevaluation of the same