Artificial Intelligence 205 (2013) 1–29

Contents lists available at ScienceDirect

Artificial Intelligence

www.elsevier.com/locate/artint

Evidential reasoning rule for evidence combination

Jian-Bo Yang *, Dong-Ling Xu

Decision and Cognitive Sciences Research Centre, The University of Manchester, Manchester M15 6PB, UK

ARTICLE INFO

Article history:
Received 26 January 2013
Received in revised form 9 September 2013
Accepted 13 September 2013
Available online 23 September 2013

Keywords:
Evidential reasoning
Belief distribution
Dempster–Shafer theory
Bayesian inference
Multiple criteria decision analysis
Information fusion

ABSTRACT

This paper aims to establish a unique *Evidential Reasoning* (*ER*) rule to combine multiple pieces of independent evidence conjunctively with weights and reliabilities. The novel concept of *Weighted Belief Distribution* (*WBD*) is proposed and extended to *WBD* with *Reliability* (*WBDR*) to characterise evidence in complement of *Belief Distribution* (*BD*) introduced in *Dempster–Shafer* (*D–S*) theory of evidence. The implementation of the orthogonal sum operation on *WBD*s and *WBDR*s leads to the establishment of the new *ER* rule. The most important property of the new *ER* rule is that it constitutes a generic conjunctive probabilistic reasoning process, or a generalised Bayesian inference process. It is shown that the original *ER* algorithm is a special case of the *ER* rule when the reliability of evidence is equal to its weight and the weights of all pieces of evidence are normalised. It is proven that Dempster's rule is also a special case of the *ER* rule when each piece of evidence is fully reliable. The *ER* rule completes and enhances Dempster's rule by identifying how to combine pieces of fully reliable evidence that are highly or completely conflicting through a new reliability perturbation analysis. The main properties of the *ER* rule are explored to facilitate its applications. Several existing rules are discussed and compared with the *ER* rule. Numerical and simulation studies are conducted to show the features of the *ER* rule.

1. Introduction

Providing an *ER* framework for evidence combination and information fusion in an *Artificial Intelligence* (*AI*) system has been an important task and attracted significant interests and efforts from communities in *AI*, computing, operational research, decision sciences, system sciences, control theory, information systems, etc. From all databases in the Web of Knowledge, the term *ER* first appeared in a paper published by *AI* in 1985 [15], although the term might well have been used in other context earlier. In their paper, Gordon and Shortliffe embraced the use of a *D–S* scheme for evidence-aggregation processes in a hypothesis space. In another paper published also in *AI* [24], it was shown that *ER* could be conducted in the same hypothesis space using a Bayesian scheme. However, the two schemes are different, and the nature and significance of their differences were investigated [20].

The *D–S* scheme is based on a frame of discernment composed of a set of propositions that are mutually exclusive and collectively exhaustive [25]. In the *D–S* scheme, basic probabilities can be assigned to not only singleton propositions but also any of their subsets, thereby allowing a piece of evidence to be profiled by a *BD* defined on the power set of the frame of discernment. *BD* is regarded as the most natural and flexible generalisation of conventional probability distribution in the sense that the former allows inexact reasoning at whatever level of abstraction [15] and on the other hand reduces to the latter if basic probabilities are assigned to singleton propositions only. It is in this context that *D–S* theory is claimed to generalise Bayesian inference [25]. Indeed such generalisation differentiates between ignorance (or lack of knowledge,

* Corresponding author.
E-mail addresses: jian-bo.yang@mbs.ac.uk (J.-B. Yang), ling.xu@mbs.ac.uk (D.-L. Xu).

0004-3702/$ – see front matter
http://dx.doi.org/10.1016/j.artint.2013.09.003

as defined in Section 2.1) and equal likelihood, and does not assume evidence partially in favour of a proposition to be construed as evidence against the same proposition through the commitment of the remaining belief to its negation [15,20]. These attractive features have motivated the use of *D–S* theory in many areas like knowledge-based system [22,8], pattern recognition [6], information fusion [21,30,14], *Multiple Criteria Decision Analysis* (*MCDA*) and risk analysis [38,40,2,41,37,33,42].

The kernel of *D–S* theory is Dempster's rule [4,5,25], which is rooted in probability theory and constitutes a conjunctive probabilistic inference process. Indeed it generalises Bayes' rule and was promoted as the sole *Evidence Combination Rule* (*ECR*) to combine evidence in the *D–S* framework originally [25]. As it stands however, Dempster's rule has no definition and cannot be applied in a special case when two pieces of evidence are in complete conflict, i.e. with each supporting different propositions. This lack of definition has led to a counter-intuitive problem when the rule is used to combine evidence in high (or near complete) conflict [46,23,16,32]. Another concern about Dempster's rule is that it assumes that each piece of evidence is fully reliable and can veto any proposition, as discussed in Section 2.2 in detail. This means that if a piece of evidence does not support a proposition at all, that proposition will be ruled out completely. In other words, Dempster's rule accumulates consensus support only and rejects a proposition completely if it is opposed by any evidence, no matter what support it may get from any other evidence. While this may be acceptable in special cases, general situations are that multiple pieces of evidence are of a compensatory nature and each play limited roles or have various degrees of reliabilities in support for and opposition against propositions [41,28].

Since the counter-intuitive problem of Dempster's rule was identified over three decades ago, a plethora of alternative *ECR*s have been developed and reviewed in the literature [16,17,14], each with its own merits as well as drawbacks. In this paper, it is not intended to provide another full review of these existing rules. Instead, typical *ECR*s will be briefly compared to emphasise the motivation of this research. Existing alternative *ECR*s are aimed to replace Dempster's rule for addressing the counter-intuitive problem and can be differentiated on the basis of how they deal with conflict among evidence [14]. Three typical views can be found in the literature: (1) allocating conflicting beliefs to the frame of discernment as global ignorance [36], (2) allocating conflicting beliefs to a subset of relevant focal propositions as local ignorance [9] or redistributing it among focal propositions locally [27], and (3) modifying initial belief function to better represent original information without modifying Dempster's rule [16].

One common observation of the alternative *ECR*s is that they are non-probabilistic in the sense that they change the specificity of evidence in basic probability assignment and/or do not constitute a Bayesian inference process when used to combine probability information. It is therefore difficult to interpret the results generated by using the alternative *ECR*s. A critical question then arises as to whether it is meaningful to replace Dempster's rule in situations where it is applicable, or whether it is sufficient to identify rules to combine pieces of highly or completely conflicting evidence. A more general question is how to combine pieces of evidence with various weights and reliabilities that have different meanings. The importance of a piece of evidence reflects a decision maker's preferences over the evidence, which is subjective, depending on who makes the judgement when using the evidence. On the other hand, reliability is used to measure the quality of a piece of evidence objectively, which is the inherent property of the information source where the evidence is generated, and is independent of who may use the evidence [28].

This paper is aimed to address the above questions and establish a unique rule, referred to as *Evidential Reasoning* rule, or *ER* rule for short, to combine multiple pieces of independent evidence for generating their joint support for a proposition. A piece of evidence is said to be independent if the information it carries does not depend on whether other evidence is known or not. This research is also motivated to investigate the rationale and foundation of the *ER* approach, which was developed to support *MCDA* of a quantitative and qualitative nature under uncertainty by applying *D–S* theory [38,40,41,43], to generalise it for evidence combination in general.

In the *ER* approach [41], a basic probability mass is generated by multiplying the degree of belief by the weight of evidence. The *Basic Probability Assignment* (*bpa*) scheme of the *ER* approach ensures that the residual support left uncommitted due to the weight of evidence is made assignable to any singleton propositions and the frame of discernment, depending upon what propositions other evidence supports. In *D–S* theory, the residual support is assigned to a specific proposition: the frame of discernment [25]. This specific assignment does not differentiate between ignorance and the residual support, whilst the former is an intrinsic property of the evidence and the latter reflects its extrinsic feature related to its relative importance compared with other evidence. This indiscrimination changes the specificity of evidence, leading to a dilemma that even if all pieces of evidence point precisely and unambiguously to a proposition their combined support for the proposition generated using the Dempster's rule will still be imprecise or incomplete.

In this paper, the new *ER* rule with evidence weight considered (or *ER* rule with weight in short) is first established by generalising the above *bpa* scheme of the *ER* approach. The first step of the *ER* rule is to construct a new *WBD* as the counterpart of *BD* for a piece of evidence to cater for its extrinsic feature of relative importance. The second step is to implement the orthogonal sum operation to combine the *WBD*s of multiple pieces of independent evidence. *WBD* is then extended to take into account both weight and reliability by constructing a new *WBD* with *Reliability* (*WBDR*). The new *ER* rule with both weight and reliability considered results from implementing the orthogonal sum operation on *WBDR*s. No specificity of any evidence is changed in the process of constructing *WBD* or *WBDR*. The *ER* rule thus constitutes a generic conjunctive probabilistic reasoning process to combine pieces of independent evidence with various weights and reliabilities.

Since it is based on the orthogonal sum operation, the *ER* rule is inherently associative and commutative, meaning that it can be used to combine multiple pieces of evidence in any order without changing the final results. In this paper, it is