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The complexity of mixed multi-unit combinatorial auctions: Tractability under structural and qualitative restrictions

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ABSTRACT

Mixed multi-unit combinatorial auctions (MMUCAs) are extensions of classical combinatorial auctions (CAs) where bidders trade transformations of goods rather than just sets of goods. Solving MMUCAs, i.e., determining the sequences of bids to be accepted by the auctioneer, is computationally intractable in general. However, differently from classical combinatorial auctions, little was known about whether polynomial-time solvable classes of MMUCAs can be singled out on the basis of their characteristics. The paper fills this gap, by studying the computational complexity of MMUCA instances under structural and qualitative restrictions, which characterize interactions among bidders and types of bids involved in the various transformations, respectively.

1. Introduction

Mixed multi-unit combinatorial auctions (MMUCAs) are extensions of classical combinatorial auctions (CAs) where participants are allowed to bid not only on bundles of goods to buy, but also on bundles of goods to sell and of transformations of goods [1].

These mechanisms are particularly useful in the context of automatizing *supply chain formation*, where production processes often emerge as the result of complex interactions among producers and consumers [2]. Indeed, in these contexts, the auctioneer wants to obtain certain products based on the goods she initially owns, by exploiting a production process possibly involving further goods to be acquired from suppliers or to be obtained via transformations operating on the goods currently available to her.

Example 1.1. Consider the supply chain associated with the production of bicycles, which is illustrated in Fig. 1 according to an intuitive graphical notation where goods are represented as ovals, transformations as boxes, and where arrows indicate inputs and outputs of the various transformations. Assume that the auctioneer is presented with 6 different bids over the singleton sets of transformations $\{t_1\}, \{t_2\}, \ldots, \{t_6\}$, whose associated prices are then reported in the boxes as well.

The assembly of a bicycle from its constituents (i.e., frame, brakes, drive train, front wheel, back wheel, seat, and handlebars) is thus offered to the auctioneer through the bid over $\{t_6\}$, which is sold for \$10. However, the auctioneer owns only a subset of such constituents (i.e., frame and brakes), plus two goods (i.e., chain and chainring) that are not immediately exploitable by t_6. The auctioneer has therefore to ask suppliers for the missing goods (i.e., front wheel, back wheel, seat, and handlebars), which are made available trough the bids over $\{t_3\}$, $\{t_4\}$, and $\{t_5\}$, for \$10, \$17, and \$8, respectively. Note that t_4 incidentally produces a dynamo, which is not part of the bicycle the auctioneer is willing to produce. Finally, note that the auctioneer

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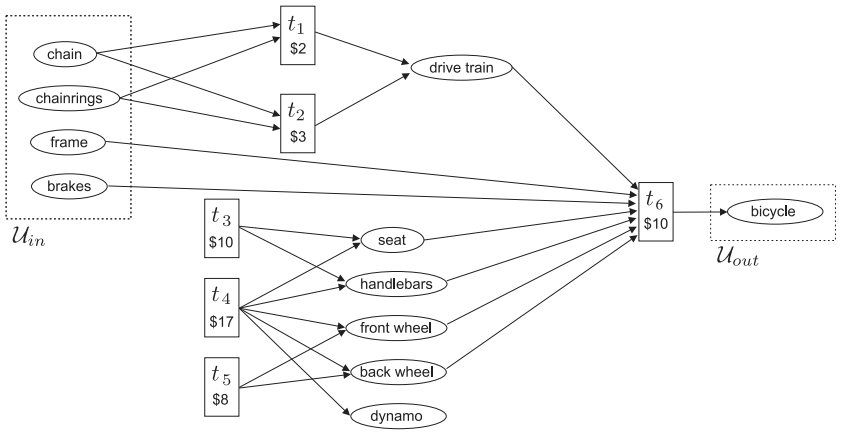

Fig. 1. Mixed multi-unit combinatorial auction in Example 1.1.

needs to accept one further bid (either over $\{t_1\}$ for \$2, or over $\{t_2\}$ for \$3) to assemble the chain and the chainrings into the drive train, in order for the latter to be taken as input by t_6. ◁

A solution to a MMUCA instance is, roughly, any set of bids whose transformations can be arranged in a sequence allowing the auctioneer to produce the desired goods. Among all possible solutions, in the Winner-Determination problem we are interested in singling out those having the minimum total cost or, equivalently, guaranteeing the maximum possible revenue. For instance, it is easily seen that the minimum production cost in Example 1.1 is \$29, which is witnessed by the sequence of transformations t_1, t_4, t_6.

The Winner-Determination problem for MMUCA instances has intensively been studied in recent years, by extending to this novel setting several results originally conceived for classical CAs. In particular, languages have been defined and analyzed which allow bidders to compose (atomic) bids in a natural and intuitive way [1], and motivated by their intractability (formally, NP-hardness), solution approaches have been proposed (see, e.g., [3]) that well-behave on realistic scenarios [4]. Differently from classical CAs, however, little was known about whether polynomial-time solvable classes of MMUCAs can be singled out based on the structural and topological properties of the instances at hand. As a matter of fact, by focusing on the kinds of interactions among bidders that are likely to occur in practice, classes of instances over which Winner-Determination is tractable—called "islands of tractability" in the literature—have been identified for classical CAs (such as *structured item graphs* [5] or *bounded hypertree-width dual hypergraphs* [6]). However, none of these results had a counterpart in the case of MMUCAs.

The aim of this paper is precisely to fill this gap, by depicting a clear and complete picture of the frontier of tractability for MMUCA instances, with respect to both *qualitative* and *structural* parameters. In particular, note that the existence of a solution is not guaranteed in the case of MMUCAs. For instance, in Example 1.1, if the auctioneer does not initially own the frame, then no solution exists at all. Therefore, checking for the feasibility of the production process is an important and peculiar source of complexity for MMUCA instances and, accordingly, attention will be focused not only on the Winner-Determination but also on the Feasibility problem of deciding whether a given instance admits a solution at all (no matter of its cost).

Our contribution can be summarized as follows:

(1) We chart the tractability frontier of the Feasibility problem for MMUCAs under *qualitative* restrictions, i.e., under restrictions characterizing the types of bids in terms of the variety and quantity of goods involved in the various transformations. The analysis is carried out over three different bidding languages:
- *Atomic bids*, where each bid is just defined over one set of transformations—this is the building block of the following two languages;
- OR-*language*, where bidders submit sets of atomic bids and accept to implement any combination of them for the sum of their prizes; and
- XOR-*language*, where each bidder accepts to implement at most one atomic bid from the set of hers submitted atomic bids.

In particular, for the above bidding languages, we analyze the scenario where each underlying atomic bid is defined over a set containing one transformation only (as in Example 1.1), as well as the more general case where each atomic bid is defined over an arbitrary set of transformations.

(2) We study the complexity of Feasibility under *structural* restrictions of the networks originating from bidder interactions, motivated by the fact that many NP-hard problems in different application areas are known to be efficiently solvable when restricted to instances that can be modeled via (nearly)acyclic instances. Surprisingly, bad news emerged from