Artificial Intelligence 175 (2011) 1194–1222

Contents lists available at ScienceDirect

Artificial Intelligence

www.elsevier.com/locate/artint

Representing preferences using intervals

Meltem Öztürk [a,*], Marc Pirlot [b], Alexis Tsoukiàs [c]

[a] CRIL-CNRS, Université d'Artois, 62307 Lens, France
[b] Faculté Polytechnique de Mons, UMONS, Belgium
[c] CNRS-Lamsade, Université Paris Dauphine, Paris, France

ARTICLE INFO

Article history:
Received 28 February 2009
Received in revised form 26 July 2010
Accepted 26 July 2010
Available online 2 December 2010

Keywords:
Preference modeling
Interval representation
Intransitivity
Thresholds

ABSTRACT

In this paper we present a general framework for the comparison of intervals when preference relations have to established. The use of intervals in order to take into account imprecision and vagueness in handling preferences is well known in the literature, but a general theory on how such models behave is lacking. In the paper we generalize the concept of interval (allowing the presence of more than two points). We then introduce the structure of the framework based on the concept of relative position and component set. We provide an exhaustive study of 2-point and 3-point intervals comparison and show the way to generalize such results to n-point intervals.

1. Introduction

Dealing with preferences is an important issue in many fields including Computer Science and Artificial Intelligence (see [9,11,18]). In general, preferences are represented by binary relations defined on a set A (finite or infinite) of alternatives to be compared or evaluated. The classical theory of preference modeling considers two relations, strict preference P and indifference I (for a more general presentation on preference modeling see [28,31]). Such a representation admits the existence of a complete preference structure, i.e. the decision maker is supposed to be able to compare any pair of alternatives (for all objects a and b in A, aPb or bPa or aIb holds). Other types of preference structures have been studied in the literature, either partial ones [15,16,41] and/or admitting more relations [10,30,33,45,40,42,43].

In this paper we focus on complete preference structures defined on a finite set A admitting two binary relations P and I. P is assumed to be an asymmetric relation and I is defined as the symmetric complement of P. The union of P and I is denoted by R (by construction R is complete and reflexive and the relation $P \cap I$ is empty) and the affirmation aRb holds if and only if "a is at least as good as b". Among others, completeness is a crucial property in order to obtain a numerical representation of the preference structure. In fact, exploiting preferences requires naturally a model and a majority of existing models are quantitative ones, the quantification of preferences rendering easier the search for optimal or near-optimal decisions. In this perspective, a number of contributions in decision theory are based on the representational theory of measurement, formalized by Scott and Suppes [35] and presented in details in the three-volume set by Krantz et al. [20], Suppes et al. [38] and Luce et al. [22]. Generally speaking representation theorems constitute a crucial aspect in handling preferences. Consider a recommender system trying to understand the preference structure of a user through a number of preferential statements. If the user claims that a is indifferent to b and this indifferent to c, but a is better than c, then we know that we need to use a numerical representation using intervals instead of single numbers in order to handle such preferences. On the other hand consider an agent who is trying to compare objects whose values (on some

* Corresponding author.
E-mail address: ozturk@lamsade.dauphine.fr (M. Öztürk).

0004-3702/$ – see front matter
doi:10.1016/j.artint.2010.11.013

attribute) are expressed imprecisely: a is between 10 and 12, b is between 11 and 14, c is between 13 and 15. How do we compare such objects? There are preference structures (in this case interval orders) that allow to establish a preference among a, b and c.

Linear orders and weak orders are well known complete structures. A linear order consists of an arrangement of objects from the best one to the worst one without any ex aequo while a weak order defines the indifference relation as an equivalence relation (reflexive, symmetric and transitive). A weak order is indeed a total order of the equivalence (indifference) classes of A. Such preference structures have a limited representation capacity. In particular, a well known problem with linear orders or weak orders is that the associated indifference relation is necessarily transitive and such a property may be violated in the presence of thresholds as in the famous example given by Luce [21] on a cup of coffee. Different structures have been introduced for handling such cases. Indeed, in contrast to the strict preference relation, the indifference relation induced by such structures is not necessarily transitive. Semiorders may form the simplest class of such structures and they appear as a special case of interval orders. The axiomatic analysis of what we now call interval orders has been given by Wiener [47], then the term "semiorders" has been introduced by Luce [21] and many results about their representations are available in the literature (for more details see [16,29]). Fishburn [17] has distinguished nine non-equivalent ordered sets defined as a generalization of semiorders (using preference structures allowing only strict preference and indifference). These are interval orders, split semiorders, split interval orders, tolerance orders, bitolerance orders, unit tolerance orders, bisemiorders, semitransitive orders and subsemitransitive orders.

The use of simple numbers appears insufficient for the representation of ordered sets having a non-transitive indifference relation. For instance, the numerical representation of an interval order makes use of intervals in a way that each alternative is represented by an interval (with a uniform length in the case of semiorders) and is said preferred to another alternative if and only if its associated interval is completely to the right of the other's interval. It is known that a majority of the structures belonging to the classification given by Fishburn [17] has a numerical representation using intervals, possibly with additional interior points.

However, the literature lacks a systematic study of such structures. Indeed as soon as we allow to compare "intervals" we can accept several different ways to do so. Just consider the case of the well known model of interval order where strict preference corresponds to the case where an interval is "completely to the right" (in the sense of the reals) of the other one. We could also consider as strict preference the case where an interval is just to the right of the other one despite having a non-empty intersection. This idea has led to the study of structures such as *tolerance order* and *bitolerance order* [7,17]. In a tolerance order for instance, a single point inside the interval determines a tolerance threshold: an object a is preferred to an object b if the interval (with one interior point) associated with it either lies completely to the right of the interval (with one interior point) representing b or the left endpoint of a lies between the interior point and the right endpoint of the b interval. So, strict preference tolerates some overlap of the intervals, in contrast with the original interval order. A similar idea, using two interior points instead of one, is implemented in bitolerance orders.

Obviously the number of possibilities for defining intuitively interesting preference structures increases dramatically with the number of "intermediate points" within an interval so that we need a general framework within which studying them. In this paper we propose such a general framework for the study of preference structures to be used when we compare intervals with distinguished intermediate points. Our objective is to propose a systematic analysis of such structures and their numerical representations. We generalize the concept of interval allowing, besides the two extreme points of an interval, the existence of a certain number of intermediate points. We call such intervals n-point intervals. The rules for comparing these intervals are supposed to satisfy some intuitive hypotheses that we define at the beginning of our study.

Besides pursuing the study of the comparison of intervals and their extensions in the spirit of the research initiated in the theory of ordered sets and that of relational preference models, our models may also allow for an interpretation related to the comparison of fuzzy numbers in two different ways:

- How to use preference relations of our framework in order to compare fuzzy numbers (or fuzzy intervals)?
- Are there some links between preference relations analyzed in our framework and some fuzzy interval comparison indices proposed in the literature?

The following section is devoted to the first question. The results concerning the second one are presented after the sections related to 2-point and 3-point intervals; two special types of n-point intervals on which we make a special focus on this article. In fact 2-point intervals correspond to a special case of fuzzy intervals, generally called "crisp intervals" (the degree of membership of all the points of the interval is 1). Most comparison indices introduced in the literature are based on the form of the membership functions and on measures of surfaces between these functions and the horizontal axis. Hence they are much dependent on the precise membership value assigned to each point in the support of the fuzzy interval. In contrast, our rules for comparing n-point intervals are completely ordinal and provide only crisp comparisons. For that reason we decided to focus our attention on comparison indices of ordinal type. Section 6.2 is devoted to this analysis and follows the section where we present our results on 2-point intervals. At the end of the section devoted to 3-point intervals, we also comment on the relationship between our rules and ordinal comparisons of fuzzy numbers (see Section 7.6).

The rest of the paper is organized as follows: Section 3 introduces basic notions, Section 4 presents hypotheses on the comparison rules and numerical representations that we can create in our framework. Section 5 shows some general results