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An approach to fuzzy soft sets in decision making based on grey relational analysis and Dempster–Shafer theory of evidence: An application in medical diagnosis

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ABSTRACT

Objective: The existing methods of fuzzy soft sets in decision making are mainly based on different kinds of level soft sets, and it is very difficult for decision makers to select a suitable level soft set in most instances. The goal of this paper is to present an approach to fuzzy soft sets in decision making to avoid selecting a suitable level soft set and to apply this approach to solve medical diagnosis problems.
Methods: This approach combines grey relational analysis with the Dempster–Shafer theory of evidence. It first utilizes grey relational analysis to calculate the grey mean relational degree, by which we calculate the uncertain degree of various parameters. Then, on the basis of the uncertain degree, the suitable basic probability assignment function of each independent alternative with each parameter can be obtained. Next, we apply Dempster–Shafer rule of evidence fusion to aggregate these alternatives into a collective alternative, by which these alternatives are ranked and the best alternative is obtained. Finally, we compare this approach with the mean potentiality approach.
Results: The results demonstrate the effectiveness and feasibility of this approach vis-a-vis the mean potentiality approach, Feng's method, Analytical Hierarchy Process and Naive Bayes' classification method because the measure of performance of this approach is the same as that of the mean potentiality approach, and the belief measure of the whole uncertainty falls from the initial mean 0.3821 to 0.0069 in an application of medical diagnosis.
Conclusion: An approach to fuzzy soft sets in decision making by combining grey relational analysis with Dempster–Shafer theory of evidence is introduced. The advantages of this approach are discussed. A practical application to medical diagnosis problems is given.

1. Introduction

In 1999, Molodtsov [1] initiated soft set theory as a new mathematical tool for dealing with uncertainties. Unlike traditional tools for dealing with uncertainties, such as the theory of probability, the theory of fuzzy sets [2], and the theory of rough sets [3], the advantage of soft set theory is that it is free from the inadequacy of the parametrization tools of those theories. Recently, there has been rapid growth in interest in soft set theory. Many efforts have been devoted to further generalizations and extensions of soft sets. Maji et al. [4,5] studied soft set theory and defined fuzzy soft sets by combining soft sets with fuzzy sets. The study of hybrid models combining soft sets or fuzzy soft sets with other mathematical structures and new operations is emerging as an active research topic of soft set theory [6,7]. Aktas et al. [8] initiated soft groups. Jun applied soft set theory to BCK/BCI-algebras [9–11]. Feng et al. [12] defined soft semirings and established a connection between soft sets and semirings. Li et al. [13] discussed relationships among soft sets, soft rough sets and topologies. Li et al. [14] investigated soft coverings. With the rapid development of soft set theory, applications of soft sets are progressing rapidly as well. Soft set theory has been proved to be useful in many kinds of fields, such as data analysis [15], forecasting [16], simulation [17], rule mining [18] and decision making [17–21].

Recently, applications of soft sets in decision making have attracted more and more attention. To cope with fuzzy soft sets-based decision making problems, Cağman et al [20,23] presented

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soft matrix theory and the uni-int decision making approach, which selected a set of optimum elements from different alternatives. Moreover, Roy and Maji [22] discussed score value as the evaluation basis for finding an optimal choice object in fuzzy soft sets. However, Kong et al. [24] argued that the Roy-Maji method was incorrect by using a counter example, and they combined two evaluation bases of choice value and score value by grey relational analysis to make decisions in a fuzzy soft set. Later, Feng et al. [25] applied level soft sets to discuss fuzzy soft sets-based decision making. Jiang et al. [26] generalized the adjustable approach to fuzzy soft sets- based decision making and presented an adjustable approach to intuitionistic fuzzy soft sets-based decision making by using level soft sets of intuitionistic fuzzy soft sets. Based on Feng' works, Basu et al. [27] further investigated the previous methods to fuzzy soft sets in decision making and introduced the mean potentiality approach, which was showed more efficient and more accurate than the previous methods. In addition, Xiao et al. [28] gave a new evaluation method based on Dempster–Shafer generalized fuzzy soft sets and applied this method in medical diagnosis problems.

For methods for decision making in fuzzy soft sets, there is no unique or uniform criterion for the selection. Although researchers focus on approaches to the evaluation basis such as choice value, score value, or other evaluation bases to construct different kinds of level soft sets, the same decision problem may obtain different results from various evaluation bases and suitable level soft sets. As a result, it is difficult to judge which result is right, and we do not know whose level soft set should be chosen for selecting the optimal choice object. The key to this problem is how to reduce subjectivity and uncertainty when we choose the decision-making method. Then, it is necessary to pay attention to this issue, and we should try to study another approach to fuzzy soft sets-based decision making to avoid the above problems.

Grey relational analysis is an important method to reflect uncertainty in grey system theory initiated by Deng [29], which is utilized for generalizing estimates under small samples and uncertain conditions. It has been successfully applied to solving decision-making problems [28–31].

Dempster–Shafer theory of evidence proposed by Dempster [34] originated from the upper and lower probabilities, and then, Shafer [35] refined and developed it. It is a powerful method for combining accumulative evidence of changing prior opinions in the light of new evidence. Compared to probability theory, Dempster–Shafer theory of evidence captures more information to support decision making by identifying uncertain and unknown evidence. It provides a mechanism to derive solutions from various forms of vague evidence without having much prior information. It is well known that Dempster–Shafer theory of evidence has been successfully applied in many fields such as intelligent medical diagnosis [28,36], knowledge reduction [37], fault diagnosis [38], multi-class classification [39], supplier selection [31], information fusion of water quality measurement sensors [40], etc. However, there have been few studies completed for soft sets-based decision making by combining Dempster–Shafer theory of evidence with grey relational analysis; in particular there has been no work completed for fuzzy soft sets-based decision making by rough set approaches. Therefore, it is meaningful to explore the fuzzy soft sets in decision making by combining grey relational analysis and Dempster–Shafer theory of evidence, and this enables decision makers to take advantage of both methods' merits and allows evaluation experts to deal with uncertainty and risk confidently. Moreover, this not only allows us to avoid the above problem of selecting the suitable level soft set to obtain choice values and score values but also helps to reduce the uncertainty caused by people's subjective cognition so as to raise the choice decision level.

In this paper, we present an approach to fuzzy soft sets in decision making by combining grey relational analysis and Dempster–Shafer theory of evidence. Moreover, we also propose an illustrative example to interpret the basic principal and the application of this approach to fuzzy soft sets-based decision making. Finally, by comparing the newly proposed approach with the mean potentiality approach to fuzzy soft sets-based decision making, we show the effectiveness of the newly proposed approach.

The remaining part of this paper is organized as follows. In Section 2, we present some concepts about fuzzy soft sets and Dempster–Shafer theory of evidence. In Section 3, we recall the mean potentiality approach to fuzzy soft sets in decision making. In Section 4, we apply grey relational analysis to determine the uncertain degree of each parameter, by which suitable Mass functions with respect to each parameter are constructed. In addition, we use a Dempster–Shafer rule of evidence combination to make the decision. Moreover, an example is employed to illustrate our approach. In Section 5, an application in medical diagnosis is given, and we compare our approach with the mean potentiality approach, which shows the feasibility of our approach. In Section 6, we conclude this paper and highlight the prospects for potential future development.

2. Preliminaries

Throughout this paper, U denotes an initial universe, E denotes the set of all possible parameters, and 2^U denotes the family of all subsets of U. We only consider the case in which U and E are both nonempty finite sets. In this section, we briefly recall some basic concepts about fuzzy soft sets, the measure of performance of a method and Dempster–Shafer theory of evidence.

2.1. Fuzzy soft sets

Definition 2.1. ([1]) Let $A \subseteq E$. A pair (F, A) is called a soft set over U, if F is a mapping defined by $F: A \to 2^U$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of (F, A).

Definition 2.2. ([5]) Let I^U be the set of all fuzzy subsets of U. Let $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F: A \to I^U$.

It is easy to see that every soft set may be considered a fuzzy soft set [25]. Let $x \in U$ and $e \in A$. $F(e)$ is a fuzzy subset of U, and it is called a fuzzy value set of parameter e. If $F(e)$ is a crisp subset of U, then (F, A) is degenerated to be the standard soft set. Let $F(e)(x)$ denote the membership value that object x holds parameter e; then $F(e)$ can be written as a fuzzy set such that $F(e) = \{x/F(e)(x) \mid x \in U\}$.

Example 2.3. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $A = \{e_1, e_2, e_3, e_4\}$. Let (F, A) be a fuzzy soft set over U, defined as follows

$$F(e_1) = \{h_1/1, h_2/1, h_3/0.2, h_4/0.3, h_5/1, h_6/0.5\},$$

$$F(e_2) = \{h_1/1, h_2/0.1, h_3/0.3, h_4/0.2, h_5/0.1, h_6/1\},$$

$$F(e_3) = \{h_1/0.1, h_2/0.4, h_3/1, h_4/1, h_5/0, h_6/0.1\},$$

$$F(e_4) = \{h_1/0.1, h_2/0.3, h_3/1, h_4/1, h_5/0, h_6/0.9\}.$$

Then, (F, A) is described by the following Table 1.

Definition 2.4. Let $((F, A)$ and (G, B) be two fuzzy soft sets. Then, "(F, A) AND (G, B)" is a fuzzy soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \widetilde{\cap} G(\beta)$ for $\alpha \in A$ and $\beta \in B$, where $\widetilde{\cap}$ is the operation "fuzzy intersection" of two fuzzy sets.