Artificial Intelligence in Medicine 63 (2015) 171–179

Contents lists available at ScienceDirect

Artificial Intelligence in Medicine

journal homepage: www.elsevier.com/locate/aiim

Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses

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ARTICLE INFO

Article history:
Received 3 August 2014
Received in revised form
11 December 2014
Accepted 18 December 2014

Keywords:
Simplified neutrosophic set
Single valued neutrosophic set
Interval neutrosophic set
Cosine similarity measure
Medical diagnosis

ABSTRACT

Objective: In pattern recognition and medical diagnosis, similarity measure is an important mathematical tool. To overcome some disadvantages of existing cosine similarity measures of simplified neutrosophic sets (SNSs) in vector space, this paper proposed improved cosine similarity measures of SNSs based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures. Then, weighted cosine similarity measures of SNSs were introduced by taking into account the importance of each element. Further, a medical diagnosis method using the improved cosine similarity measures was proposed to solve medical diagnosis problems with simplified neutrosophic information.

Materials and methods: The improved cosine similarity measures between SNSs were introduced based on cosine function. Then, we compared the improved cosine similarity measures of SNSs with existing cosine similarity measures of SNSs by numerical examples to demonstrate their effectiveness and rationality for overcoming some shortcomings of existing cosine similarity measures of SNSs in some cases. In the medical diagnosis method, we can find a proper diagnosis by the cosine similarity measures between the symptoms and considered diseases which are represented by SNSs. Then, the medical diagnosis method based on the improved cosine similarity measures was applied to two medical diagnosis problems to show the applications and effectiveness of the proposed method.

Results: Two numerical examples all demonstrated that the improved cosine similarity measures of SNSs based on the cosine function can overcome the shortcomings of the existing cosine similarity measures between two vectors in some cases. By two medical diagnoses problems, the medical diagnoses using various similarity measures of SNSs indicated the identical diagnosis results and demonstrated the effectiveness and rationality of the diagnosis method proposed in this paper.

Conclusions: The improved cosine measures of SNSs based on cosine function can overcome some drawbacks of existing cosine similarity measures of SNSs in vector space, and then their diagnosis method is very suitable for handling the medical diagnosis problems with simplified neutrosophic information and demonstrates the effectiveness and rationality of medical diagnoses.

1. Introduction

Due to the increased volume of information available to physicians from modem medical technologies, medical diagnosis contains a lot of incomplete, uncertainty, and inconsistent information, which is essential information about medical diagnosis problems. A symptom usually implies a lot of incomplete, uncertainty, and inconsistent information for a disease, which characterizes a relation between symptoms and diseases. Thus we work with the uncertainties and inconsistencies to lead us to proper decision making in medicine. In most of the medical diagnosis problems, there exist some patterns, and then the experts make a decision based on the similarity between unknown sample and the basic diagnosis patterns. In some practical situations, there is the possibility of each element having different truth-membership, indeterminacy-membership, and falsity-membership functions. Therefore, Smarandache [1] originally proposed the concept of a neutrosophic set from philosophical point of view. A neutrosophic set A in a universal set X is characterized independently by

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http://dx.doi.org/10.1016/j.artmed.2014.12.007
0933-3657/

a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. However, the domain of definition and range of the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a neutrosophic set A is the non-standard unit interval $]^-0, 1^+[$, it is only used for philosophical applications, especially when a distinction is required between absolute and relative truth/falsity/indeterminacy. To easily use in technical applications of the neutrosophic set, the domain of definition and range of $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be restrained to the normal standard real unit interval [0, 1]. As a simplified form of the neutrosophic set, a simplified neutrosophic set (SNS) in [2] is an appropriate choice as it easily expresses and deals with incomplete, uncertainty, and inconsistent information in real science and engineering fields. SNSs include single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs) and a generalization of classic sets, fuzzy sets (FSs) [3], intuitionistic fuzzy sets (IFSs) [4] and interval-valued intuitionistic fuzzy sets (IVIFSs) [5]. However, FSs, IFSs and IVIFSs cannot represent and handle uncertainty and inconsistent information [1]. Then, similarity measure is not only an important mathematical tool in pattern recognition, medicine diagnosis, and decision making but also an important research topic in the neutrosophic theory. Various similarity measures have been proposed by some researchers. Broumi and Smarandache [6] defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. Majumdar and Samanta [7] introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [8] also presented the Hamming and Euclidean distances between INSs and their similarity measures and applied them to multiple attribute decision-making problems with interval neutrosophic information. Ye [9] further proposed the distance-based similarity measure of SVNSs and applied it to group decision making problems with single valued neutrosophic information. Furthermore, Ye [2] proposed three vector similarity measures for SNSs, including the Jaccard, Dice, and cosine similarity measures for SVNSs and INSs, and applied them to multicriteria decision-making problems with simplified neutrosophic information. Till now, existing similarity measures for neutrosophic sets are scarcely applied to medical diagnosis problems. However, the cosine similarity measures defined in vector space [2,10] have some drawbacks in some situations. For instance, they may produce no defined (unmeaningful) phenomena or some results calculated by the cosine similarity measures are unreasonable in some real cases (details given in Section 3). Therefore, in the situations, it is difficult to apply them to pattern recognition and medicine diagnosis. To overcome some drawbacks of existing cosine measures in [2], this paper aims to propose improved cosine similarity measures for SNSs and apply them to medical diagnosis. To do so, the rest of the article is structured as follows. In Section 2, we briefly introduce some basic concepts of SNSs. Section 3 reviews existing cosine similarity measures of SNSs in vector space and their drawbacks. Section 4 proposes improved cosine similarity measures of SNSs based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures, and investigates their properties. In Section 5, by two numerical examples we give the comparative analysis between the improved cosine similarity measures and existing cosine similarity measures for SNSs to show the effectiveness and rationality of the improved cosine measures. In Section 6, a medical diagnosis method is proposed based on the improved cosine similarity measures and is applied to medical diagnosis problems. Conclusions and further research are given in Section 7.

2. Some basic concepts of SNSs

Smarandache [1] originally presented the concept of a neutrosophic set from philosophical point of view. In a neutrosophic set A in a universal set X, its characteristic functions are expressed by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, respectively. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ is no restriction, i.e., $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

To apply a neutrosophic set to science and engineering areas, Ye [2] introduced SNS, which is a subclass of the neutrosophic set, and gave the following definition of a SNS.

Definition 1 *([2])*. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], such that $T_A(x)$: $X \to [0, 1]$, $I_A(x)$: $X \to [0, 1]$, and $F_A(x)$: $X \to [0, 1]$. Then, a simplification of the neutrosophic set A is denoted by

$$A = \{\langle x,\ T_A(x),\ I_A(x),\ F_A(x)\rangle | x \in X\},$$

which is called a SNS. It is a subclass of the neutrosophic set and includes the concepts of INS and SVNS.

On the one hand, if we only consider that the values of $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a SNS A are single points in the real standard [0, 1] instead of subintervals/subsets in the real standard [0, 1], the SNS A can be described by three real numbers in the real unit interval [0, 1]. Therefore, the sum of $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. In this case, the SNS A reduces to the SVNS A.

For two SVNSs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle | x \in X\}$, there are the following relations [11]:

(1) Complement: $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}$;
(2) Inclusion: $A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X;
(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

On the other hand, if we only consider three membership degrees in a SNS A as the subunit interval of the real unit interval [0, 1], the SNS can be described by three interval numbers in the real unit interval [0, 1]. For each point x in X, we have that $T_A(x) = [\inf T_A(x), \sup T_A(x)]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)]$, $F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0, 1]$, and $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$ for any $x \in X$. In this case, the SNS A reduces to the INS A.