Available online at www.sciencedirect.com

ScienceDirect

Cognitive Systems Research 40 (2016) 18–34

Cognitive Systems RESEARCH

www.elsevier.com/locate/cogsys

Quantification of human confidence in functional relations

Action editor: Peter Erdi

Pooyan Doozandeh *

Vishgaahi Alley, Motahhari St., Rasht, Gilan 41846-33698, Iran

Received 22 October 2015; received in revised form 18 January 2016; accepted 9 February 2016
Available online 17 February 2016

Abstract

What makes people infer that two continuous-valued entities are functionally related? Involving factors influencing human confidence in the existence of a functional link between two supposed variables has not so far been discussed in function learning literature. By examining this problem and based on relevant results from cognitive psychology, I propose a hypothesis according to which human confidence in a link between cue and criterion is affected by three factors: The difficulty of functions, the level of noise in observed data, and the sample size. Here, the formalization of this hypothesis forms a novel mathematical model of function learning which can also be used for predictions; so the resulting model receives cue-criterion pairs of a supposed relation and produces two outputs: Confidence and predicting function. In an experiment, the performance of a computational implementation of the model is compared with human data. The results show that the model is successful in tracking changes in human confidence. A close correspondence between the predictions of the model and humans was also achieved.

Keywords: Function learning; Function recognition; Confidence measurement; Mathematical model

1. Introduction

People can learn functional relations in real-world dynamic environments. This ability is used so frequently that we hardly even recognize it; it is the prerequisite of our judgments in multifarious contexts. In everyday life, we may perceive it in establishing the relationship between turning the volume knob of a radio and the change in the sound intensity of it; or the amount of coffee intake and increased wakefulness. In medical research, scientists may investigate how, for instance, a change in the amount of cigarettes smoked per day would change the probability of lung cancer or heart attack. In human sciences and economy, a simple example can be the notion that the increase in cash injection together with a decrease in production, would lead to an increase in inflation rate. In short, learning functional relations is essential in human judgment and a prerequisite for human knowledge. Establishing functional relations can also be thought of as cases of causal induction with both variables of cause and effect take continuous values.

The existence of a single mental mechanism behind all instances of human function learning is the shared assumption of cognitive psychologists. From this view, function learning is a mental system that, at first, measures the magnitude of two variables,[1] namely cue (cause) and criterion (effect); for example, the degree of the pressure of gas pedal is the value for the cue and the RPM[2] of the engine is the

* Tel.: +98 9353707289.
E-mail address: pooyan.doozandeh@gmail.com.

[1] There can be more than two variables, both for the cue and criterion. For the sake of simplicity and plotting the pairs of cue-criterion, the model and experiments in this article work with single cue tasks.

[2] Revolution per minute.

http://dx.doi.org/10.1016/j.cogsys.2016.02.001
1389-0417/

value for the criterion. In the next step, the learner would sample the supposed relation by considering certain pairs of values of the cue and at those values, measuring the corresponding values of the criterion. Having a number of cue-criterion pairs, the mental system receives them as input. The output is a mathematical function (either explicit like algebraic rules or implicit like artificial neural networks) which is fitted on input pairs, as the final judgment that is generalized over the entire possible values of the cue. This generalization of the values of criterion from observed samples of the cue is also called prediction; since the resulting function can predict the values of criterion for unobserved cues. The main subject of the literature of modeling in function learning has so far lain in the question of what theoretical system we can devise that would have the human capability in such predictions.

Before investigations on human power in prediction, I think we should first take one step back and address a more fundamental question: What makes people infer that there exists a functional (causal) link connecting two variables of cue and criterion? In other words, in light of the assumptions of cognitive psychology, if a human observer is presented with a set of cue-criterion pairs, what factors affect his/her confidence in the existence of a functional link between the cue and criterion?

By focusing on cue-criterion pairs, the present study aims for introducing a measure that can quantify human confidence in the existence of functional relations. This measure is then embedded in a novel mathematical model of function learning that can also be used for prediction. The model integrates past findings in cognitive psychology and uses the rule-based approach (Carroll, 1963; Brehmer, 1974; Koh & Meyer, 1991), combining knowledge before and after observations.

In order to assess the existence of a functional relation between the cue and criterion, the model of this research receives cue-criterion pairs as input. Then, a number of preexisting mathematical functions are fitted on input pairs and for each parameterized function if the goodness of fit is higher than a threshold, the model calculates a confidence measure. In the end, the function with the largest confidence measure is chosen. This measure is the confidence in the existence of a functional link between the cue and criterion and is then passed as the first and primary output of the model. The corresponding rule of the function is the other output that can be used for predictions.

The next section examines previous relevant works in causal induction and function learning. In Section 3, affecting factors in measuring the confidence are introduced and developed and then, the measure of confidence and the general model are formulated and presented. Section 4 explains experiments with human participants and Section 5 includes a general evaluation of the model's performance. Section 6 discusses abstract issues of function learning and mathematical modeling with respect to the problems and shortcomings of the model.

2. Background

Researches on human causal induction with binary-valued variables of cause and effect have been both older and more popular than continuous-valued causal induction (or function learning). This is in spite of the fact that for human judgment, learning, and decision making, function learning is as equally, if not more, important. Upon closer examination, it becomes obvious that these two fields have similarities and many examples of binary-valued causal induction can be thought of as examples of function learning. For example, Griffiths and Tenenbaum (2005) give an example of a binary-valued causal induction between injection of mice with a certain chemical and the expression of a particular gene. Viewed closely, it can be an example of continuous-valued causal induction in which the amount of chemical and the length of the gene can take continuous values. We can similarly think of other famous examples of causal induction like the relationship between smoking and lung cancer, or between coke consumption and diabetes.

While most research in binary-valued causal induction had been centered on assessing the strength of the relation between cause and effect, Griffiths and Tenenbaum (2005) were first to recognize the importance of questioning the existence of any causal link between cause and effect and they presented a rational model for this end. Six years later, Griffiths, Sobel, Tenenbaum, and Gopnik (2011) formulated a Bayesian model for causal induction that could both incorporate prior knowledge and address the problem of assessing the existence of a causal relation between two binary-valued variables of cause and effect.

In the literature of function learning, however, there has not been a similar attempt in creating a model that can determine the existence of a causal relation between continuous-valued variables of cause and effect. This investigation seems to be absent from both theoretical psychology and cognitive or rational modeling. For example, in the case of drinking coffee and wakefulness, how do we infer if there exists any causal link between them? And how can we quantify human confidence in the existence of the link?

Models in the history of research in function learning are usually divided in two groups. The first, which is commonly called the rule-based group of models, assumes that the task of learning a function is conducted by approximating explicit mathematical functions, or rules, on observed cue-criterion pairs (e.g., Carroll, 1963; Brehmer, 1974; Koh & Meyer, 1991; Narain, Smeets, Mamassian, Brenner, & van Beers, 2014). Similarity-based models, as the second group, argue that functions are learned associatively and novel inputs are being predicted by their degree of similarity with observed values (e.g., DeLosh, Busemeyer, & McDaniel, 1997; Busemeyer, Byun, DeLosh, & McDaniel, 1997). There have also been attempts at presenting hybrid models (McDaniel & Busemeyer, 2005; Kalish, Lewandowsky, & Kruschke,