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The truth assignments that differentiate human reasoning from mechanistic reasoning
The evidence-based argument for Lucas' Gödelian thesis

Bhupinder Singh Anand

#1003, B Wing, Lady Ratan Tower, Dainik Shivner Marg, Gandhinagar, Worli, Mumbai 400 018, Maharashtra, India

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Abstract

We consider the argument that Tarski's classic definitions permit an intelligence—whether human or mechanistic—to admit *finitary* evidence-based definitions of the satisfaction and truth of the *atomic* formulas of the first-order Peano Arithmetic PA over the domain $\mathbb{N}$ of the natural numbers in two, hitherto unsuspected and essentially different, ways: (1) in terms of *classical* algorithmic verifiabilty; and (2) in terms of *finitary* algorithmic computability. We then show that the two definitions correspond to two distinctly different assignments of satisfaction and truth to the *compound* formulas of PA over $\mathbb{N}$—$\mathcal{I}_{PA(\mathbb{N},\ SV)}$ and $\mathcal{I}_{PA(\mathbb{N},\ SC)}$. We further show that the PA axioms are true over $\mathbb{N}$, and that the PA rules of inference preserve truth over $\mathbb{N}$, under both $\mathcal{I}_{PA(\mathbb{N},\ SV)}$ and $\mathcal{I}_{PA(\mathbb{N},\ SC)}$. We then show: (a) that if we assume the satisfaction and truth of the *compound* formulas of PA are always *non-finitarily* decidable under $\mathcal{I}_{PA(\mathbb{N},\ SV)}$, then this assignment corresponds to the classical *non-finitary* standard interpretation $\mathcal{I}_{PA(\mathbb{N},\ S)}$ of PA over the domain $\mathbb{N}$; and (b) that the satisfaction and truth of the *compound* formulas of PA are always *finitarily* decidable under the assignment $\mathcal{I}_{PA(\mathbb{N},\ SC)}$, from which we may *finitarily* conclude that PA is consistent. We further conclude that the appropriate inference to be drawn from Gödel's 1931 paper on undecidable arithmetical propositions is that we can define PA formulas which—under any interpretation over $\mathbb{N}$—are algorithmically verifiable as always true over $\mathbb{N}$, but not algorithmically computable as always true over $\mathbb{N}$. We conclude from this that Lucas' Gödelian argument is validated if the assignment $\mathcal{I}_{PA(\mathbb{N},\ SV)}$ can be treated as circumscribing the ambit of human reasoning about 'true' arithmetical propositions, and the assignment $\mathcal{I}_{PA(\mathbb{N},\ SC)}$ as circumscribing the ambit of mechanistic reasoning about 'true' arithmetical propositions.
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1. Introduction

We briefly consider a philosophical challenge that arises when an intelligence—whether human or mechanistic—accepts arithmetical propositions as true under an interpretation—either axiomatically or on the basis of subjective self-evidence—*without* any specified methodology for evidencing such acceptance.[1]

For instance conventional wisdom, whilst accepting Alfred Tarski's classical definitions of the satisfiability and truth of the formulas of a formal language under an interpretation[2], *postulates* that under the classical standard

E-mail address: bhup.anand@gmail.com

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[1] For a brief recent review of such challenges, see Feferman (2006) and Feferman (2008); also Anand (2004) and Rodrigo Freire's informal essay on '*Interpretation and Truth in Cantorian Set Theory*'.

[2] Tarski (1933).

interpretation $\mathcal{I}_{PA(\mathbb{N},\ Standard,\ Classical)}$[3] of the first-order Peano Arithmetic PA[4] over the domain $\mathbb{N}$ of the natural numbers:

(i) The satisfiability/truth of the atomic formulas of PA can be assumed as *uniquely* decidable under $\mathcal{I}_{PA(\mathbb{N},\ S)}$;
(ii) The PA axioms can be assumed to *uniquely* interpret as satisfied/true under $\mathcal{I}_{PA(\mathbb{N},\ S)}$;
(iii) The PA rules of inference—Generalisation and Modus Ponens—can be assumed to *uniquely* preserve such satisfaction/truth under $\mathcal{I}_{PA(\mathbb{N},\ S)}$;
(iv) Aristotle's particularisation[5] can be assumed to hold under $\mathcal{I}_{PA(\mathbb{N},\ S)}$.

We shall argue that the seemingly innocent and self-evident assumptions of *uniqueness* in (i) to (iii)—as also the seemingly innocent assumption in (iv) which, despite being obviously *non-finitary*, is unquestioningly accepted in classical literature[6] as equally self-evident under any logically unexceptionable interpretation of the classical first-order logic FOL—conceal an ambiguity with far-reaching consequences.

The ambiguity is revealed if we note[7] that Tarski's classic definitions permit both human and mechanistic intelligences to admit *finitary*[8] evidence-based definitions of the satisfaction and truth of the *atomic* formulas of PA over the domain $\mathbb{N}$ of the natural numbers in two, hitherto unsuspected and essentially different, ways:

(1a) In terms of *classical* algorithmic verifiabilty; and
(1b) In terms of *finitary* algorithmic computability.

We shall show[9] that:

(2a) The two definitions correspond to two distinctly different assignments of satisfaction and truth to the *compound* formulas of PA over $\mathbb{N}$—say $\mathcal{I}_{PA(\mathbb{N},\ Standard,\ Verifiable)}$ and $\mathcal{I}_{PA(\mathbb{N},\ Standard,\ Computable)}$[10]; where
(2b) The PA axioms are true over $\mathbb{N}$, and the PA rules of inference preserve truth over $\mathbb{N}$, under both $\mathcal{I}_{PA(\mathbb{N},\ SV)}$ (Section 5.1) and $\mathcal{I}_{PA(\mathbb{N},\ SC)}$ (Section 6.1).

We shall then show that[11]:

(3a) If we assume the satisfaction and truth of the compound formulas of PA are always *non-finitarily* decidable under the assignment $\mathcal{I}_{PA(\mathbb{N},\ SV)}$, then this assignment defines a *non-finitary* interpretation of PA in which Aristotle's particularisation always holds over $\mathbb{N}$; and which corresponds to the classical *non-finitary* standard interpretation $\mathcal{I}_{PA(\mathbb{N},\ S)}$ of PA over the domain $\mathbb{N}$—from which only a human intelligence may *non-finitarily* conclude that PA is consistent; whilst
(3b) The satisfaction and truth of the compound formulas of PA are always *finitarily* decidable under the assignment $\mathcal{I}_{PA(\mathbb{N},\ SC)}$, which thus defines a *finitary* interpretation of PA—from which both intelligences may *finitarily* conclude that PA is consistent[12].

We shall show further that both intelligences would logically conclude that:

(4a) The assignment $\mathcal{I}_{PA(\mathbb{N},\ SC)}$ defines a subset of PA formulas that are algorithmically computable as true under the standard interpretation $\mathcal{I}_{PA(\mathbb{N},\ S)}$ if, and only if, the formulas are PA provable;
(4b) PA is not ω-consistent[13]; and
(4c) PA is categorical with respect to algorithmic computability.

Both intelligences would also logically conclude that:

(5a) Since PA is not ω-consistent, Gödel's argument in Gödel (1931) (p. 28(2))—that "Neg(17Gen r) is not κ-PROVABLE"[14]—does not yield a formally undecidable proposition in PA[15];

[3] See Appendix A. We shall refer to this henceforth as $\mathcal{I}_{PA(\mathbb{N},\ S)}$.
[4] We take this to be the first-order Peano Arithmetic defined in any standard text, such as the theory S in Mendelson (1964), p. 102.
[5] See Appendix A. Informally, we define Aristotle's particularisation as the *non-finitary* assumption that an assertion such as, 'There exists an x such that $F(x)$ holds'—usually denoted symbolically by '$(\exists x)F(x)$'—can always be validly inferred in the classical logic of predicates from the assertion, 'It is not the case that: for any given x, $F(x)$ does not hold'—usually denoted symbolically by '$\neg(\forall x)\neg F(x)$' (Hilbert & Ackermann, 1928, pp. 58–59).
[6] See Appendix A.
[7] See Anand (2012) and Anand (2015).
[8] We mean 'finitary' in the sense that "…there should be an algorithm for deciding the truth or falsity of any mathematical statement" …http://en.wikipedia.org/wiki/Hilbert's_program. For a brief review of 'finitism' and 'constructivity' in the context of this paper see Feferman (2008).
[9] cf. Anand (2012) and Anand (2015).
[10] We shall refer to these henceforth as $\mathcal{I}_{PA(\mathbb{N},\ SV)}$ and $\mathcal{I}_{PA(\mathbb{N},\ SC)}$ respectively.
[11] cf. Anand (2012) and Anand (2015).
[12] As sought by David Hilbert for the second of the twenty-three problems that he highlighted at the International Congress of Mathematicians in Paris in 1900.
[13] See Appendix A.
[14] The reason we prefer to consider Gödel's original argument (rather than any of its subsequent avatars) is that, for a purist, Gödel's remarkably self-contained 1931 paper—which neither contained, nor needed, any formal citations—remains unsurpassed in mathematical literature for thoroughness, clarity, transparency and soundness of exposition, *from first principles* (thus avoiding any implicit mathematical or philosophical assumptions), of his notion of arithmetical 'undecidability' as based on his Theorems VI and XI and their logical consequences.
[15] We note that if PA is not ω-consistent, then Aristotle's particularisation does not hold in any finitary interpretation of PA over $\mathbb{N}$. Now, J. Barkley Rosser's 'undecidable' arithmetical proposition in Rosser (1936) is of the form $[(\forall y)(Q(h,y) \rightarrow (\exists z)(z \leqslant y \wedge S(h,z)))]$. Thus his 'extension' of Gödel's proof of undecidability too does not yield a 'formally undecidable proposition' in PA, since it assumes that Aristotle's particularisation holds when interpreting $[(\forall y)(Q(h,y) \rightarrow (\exists z)(z \leqslant y \wedge S(h,z)))]$ under a finitary interpretation over $\mathbb{N}$ (Rosser, 1936, Theorem II, pp. 233–234; Kleene, 1952, Theorem 29, pp. 208–209; Mendelson, 1964, Proposition 3.32, pp. 145–146).

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