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Premise annotation in mental model construction: An ACT-R approach to processing indeterminacy in spatial relational reasoning

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Sven Brüssow [a,*], Marco Ragni [a], Matthias Frorath [a], Lars Konieczny [a], Thomas Fangmeier [a,b]

[a] *Center for Cognitive Science, University of Freiburg, Germany*
[b] *Department for Psychiatry and Psychotherapy, University Clinic Freiburg, Germany*

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Abstract

Reasoning about inference problems that allow for multiple interpretations requires maintaining intermediate representations that, if necessary, may be reconsidered at a later stage of the solution process. In that respect we describe the process of annotating premises in spatial relational reasoning that allows for the derivation of alternative representations. Furthermore, we show how ACT-R's subsymbolic processing principles substantially contribute to the underlying theoretical framework of the Preferred Mental Model Theory as they add a powerful component making precise accuracy predictions possible, a feature that in previous symbolic approaches has been neglected. In addition, we implemented and compared two strategies to investigate the persistence of the outcomes of the reasoning process. Furthermore, we examined how well data and predictions meet the central assumption that reasoning difficulty increases with the number of mental operations necessary to validate a putative conclusion.

Keywords: Spatial relational reasoning; Mental models; ACT-R

1. Introduction

Cognitive models implemented in architectures such as ACT-R (Anderson et al., 2004; Anderson, 2007) help describe various processes related to specific tasks by partitioning them into their motor, perceptual, and higher order cognitive components. They help researchers develop intuitions about cognitive demands of certain tasks, generate additional data that can be compared to human data, and, in that respect, are a valuable source guiding theory revaluation. In particular, predictions by cognitive models help quantify both within-task and between-tasks interference resulting from cognitive bottlenecks (Borst, Taatgen, & van Rijn, 2010). In the present single task context of spatial relational reasoning this addresses the issue of concurrent sub-tasks such as maintaining multiple intermediate mental representations necessary for processing indeterminate deduction problems.

The Preferred Mental Model Theory (PMMT) describes the deduction process in the context of such indeterminate descriptions and stands in the tradition of the classical Mental Model Theory (Johnson-Laird & Byrne, 1991; Johnson-Laird, 2001, 2006). Mental models are derived from information that is typically given by a set of premises. If multiple derivations result, to safely accept or reject a putative conclusion, reasoners need to test all of them. In reality, however, capacity restrictions potentially prevent the derivation of all possible mental models. In that respect, the distinction between determinate and indeterminate problems is essential: if *determinate* they allow for only one mental model; if *indeterminate* they allow

* Corresponding author. Address: Center for Cognitive Science, Friedrichstr. 50, D-79098 Freiburg, Germany. Tel.: +49 761 203 4941; fax: +49 761 203 4938.
E-mail address: sven@cognition.uni-freiburg.de (S. Brüssow).

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Table 1
Determinate (top) and indeterminate problems (bottom). Mental model denomination refers to unique (UNI), preferred (PMM) or alternative mental model (AM1, AM2). $\overset{AB}{\rightarrow}$ represents the premise "A is to the *left* of B" and $\overset{AB}{\leftarrow}$ "B is to the *right* of A." The remaining premises should be read accordingly. Arrows indicate the order of term presentation. Double curve arrows mark those terms that need to be transposed to transform the source to an alternative model.

P1	P2	P3	UNI	PMM	AM1	AM2
$\overset{AB}{\rightarrow}$ $\overset{AB}{\leftarrow}$ $\overset{AB}{\rightarrow}$ $\overset{AB}{\leftarrow}$	$\overset{BC}{\rightarrow}$ $\overset{BC}{\rightarrow}$ $\overset{BC}{\leftarrow}$ $\overset{BC}{\leftarrow}$	$\overset{CD}{\rightarrow}$ $\overset{CD}{\rightarrow}$ $\overset{CD}{\rightarrow}$ $\overset{CD}{\rightarrow}$	ABCD	∅	∅	∅
$\overset{AB}{\rightarrow}$ $\overset{AB}{\leftarrow}$ $\overset{AB}{\rightarrow}$ $\overset{AB}{\leftarrow}$	$\overset{AC}{\rightarrow}$ $\overset{AC}{\rightarrow}$ $\overset{AC}{\leftarrow}$ $\overset{AC}{\leftarrow}$	$\overset{CD}{\rightarrow}$ $\overset{CD}{\rightarrow}$ $\overset{CD}{\rightarrow}$ $\overset{CD}{\rightarrow}$	∅	$A\overset{\leftrightarrow}{BC}D$	$AC\overset{\leftrightarrow}{BD}$	ACDB

for multiple mental models (cf. Table 1). In the construction process of the single mental model resulting from determinate problems, each premise term can uniquely be assigned a position within this model. In contrast, in indeterminate problems unique term integration is not always possible. For example, processing the two successive premises "A is to the *left* of B" and "A is to the *left* of C" results in two possible positions for term "C" and, consequently, in the two different mental models "ABC" and "ACB." According to the PMMT, however, the construction costs for each possible mental model are different; the reasoning process, therefore, starts with constructing the mental model involving the lowest computational costs (Ragni, Knauff, & Nebel, 2005). In the example above, the construction of "ABC" is computationally cheaper than "ACB" as the former would not require the movement of term "B" when inserting term "C." The general assumption in the context of the PMMT is that term insertion takes place at the *first free fitting* position (fff-principle). Only if the corresponding conclusion does not hold in the resulting model (e.g., "is C to the left of B?" with respect to "ABC"), an alternative model is created (e.g., "ACB") with the respective term (e.g., "C") now at the *first fitting* position (ff-principle). This, however, involves additional movement of the term that was inserted at this position previously (e.g., "B").

In the following, we denote the single model resulting from determinate problems as "unique mental model" (UNI). In indeterminate situations, we denote the mental model that is preferred over alternatives as the "preferred mental model" (PMM). Alternatives we either denote "first alternative" (AM1), "second alternative" (AM2), or generally "alternative mental model" (AMM).

The reasoning process is commonly divided into three phases. First, in the *construction phase*, the PMM is constructed. Second, during the *inspection phase*, reasoners check if a putative conclusion holds in the PMM (Knauff, Rauh, & Schlieder, 1995; Rauh, Hagen, Schlieder, Strube, & Knauff, 2000; Rauh et al., 2005; Jahn, Knauff, & Johnson-Laird, 2007). Third, in the *variation phase* reasoners are assumed to construct AMMs by modifying the PMM if it contradicts the conclusion; dependence solely upon the PMM can lead to counter-examples being missed. For example, based on the indeterminate problems illustrated in Table 1, validating "is D to the left of B?" requires two transformations, provided that participants adhere to the predicted order of (1) PMM to AM1 and (2) AM1 to AM2.

The inference process is described algorithmically by operations starting with incremental integration of premise terms into the PMM; followed by term comparisons with respect to their relative positions within the mental model and conclusion; and, if necessary, succeeded by PMM modifications to an AMM (Ragni & Brüssow, 2011). Ragni et al. (2005) and Ragni and Knauff (2008) presented a computational model for spatial reasoning by mental models (SRM) for all three reasoning phases of the PMMT. It uses a discrete structure to represent the mental model; a mental focus to insert objects and manipulate mental models; provides model construction principles to construct the PMM, such as the fff-principle described above; and uses annotations—the encoding of premise information (cf. Rauh, 2000; Vandierendonck, Dierckx, & De Vooght, 2004). The presented cognitive model is an implementation of the SRM and its computational principles using the specific modular structure of the cognitive architecture ACT-R.

Little research has systematically investigated the processes of mental model variation. Schlieder (1999) developed a computational model for reasoning with interval relations (e.g., Knauff et al., 1995). It generates PMMs based on linear orderings of the interval endpoints and transformations on them. A limitation is that there is no explicit working memory representation. Rauh et al. (2000) were the first to describe errors of omission and commission in spatial relational reasoning with intervals in a symbolic approach. Krumnack, Bucher, Nejasmic, Nebel, and Knauff (2011) describe a computational model and cost measure they use to estimate the efficiency of the mental model construction process. These approaches, however, discount subsymbolic mechanisms that account for environmental noise or control learning and forgetting processes. Boeddinghaus, Ragni, Knauff, and Nebel (2006) were the first to develop an ACT-R model for the PMMT. It extends previous computational models by subsymbolic aspects such as the activation of objects and annotations. A limitation is—as it is developed in ACT-R 5.0—that it makes no use of an explicit module for maintaining intermediate problem state information (cf. Borst et al., 2010), a feature that was added to the current version of ACT-R (cf. Anderson, 2007). Therefore, by providing new data compared to model predictions, we investigated the potential of the newest release of ACT-R to substantiate the theoretical framework of the PMMT.

ACT-R is an architecture of cognition that implements both symbolic and subsymbolic concepts. It is empirically grounded and has successfully been used to simulate a wide range of cognitive tasks. Standing in the tradition of