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Bidding strategies in online auctions with different ending rules and value assumptions

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ABSTRACT

Compared with traditional auctions, online auctions (used by, e.g., eBay and Yahoo) have several distinguishing features, including different ending rules (hard-close and soft-close), sequential arrival of customers, and random numbers of customers, all of which make bidding behavior more complex. The phenomenon of late bidding has been reported in the literature and, although the origin of this behavior has been analyzed theoretically, it is still not clear. Here, we study both first- and second-price online auctions with either hard- or soft-close ending rules and assume either private value (PV) or common value (CV). By dividing the auction process into two stages and then using backward induction, we find that late bidding is dominant under CV, but under PV late bidding dominates only in first-price online auctions with hard-close. Moreover, for second-price online auctions the dominant strategy for customers is to report their true value immediately upon arrival under PV but near the end of the auction under CV, irrespective of ending rules. Finally, we find that the timing of customer bidding is the same for hard- and soft-close except for first-price online auctions under PV.

1. Introduction

Online auctions are widely used by eBay, Yahoo (Hong Kong, Taiwan, and Japan), Taobao, etc., and were ever used by Amazon. The main difference among online auctions used by various webs is their different ending rules: one is the so-called "hard-close" with a fixed deadline; that is, auctions end at a scheduled time, most often in seven days. The other one is the so-called "soft-close" ending rule under which auctions are automatically extended if necessary past the scheduled ending time. They only end when ten minutes pass without any bid being submitted. Auctions on eBay and Taobao use hard-close ending rule, those on Amazon used soft-close one, while those on Yahoo use both hard- and soft-close ones.

Besides the ending rule, online auctions are distinguished from traditional auctions by other features (see Easley and Tenorio 2004 for a summary). In an online auction, customers sequentially arrive at the auction website and may not know the number of customers that have preceded them or that follow them, even though each customer arrival is recorded by the website. Thus, for each customer, the number of competitors is random in online auctions. So the customers' strategy consists of determining only when and how much to bid.

Many papers on online auctions, especially empirical papers, are available in the literature (e.g., Pinker et al. 2003). Roth and Ockenfel (2002) and Bajari and Hortacsu (2003) empirically document an interesting phenomenon called "late" bidding (or last-minute bidding), which practitioners call "sniping" (that is, many bids are submitted very close to or just at the end of auctions). Late bidding is more prevalent in hard-close auctions than in soft-close auctions. The authors also present several possible explanations for the phenomenon of late bidding.

Many authors have theoretically analyzed late bidding. Using the rate equation approach, Yang and Kahng (2006) demonstrate that late bidding is a rational and effective strategy for a hard-close auction. However, Chiang and Kung (2005) find that late bidding is used only when bids are intense. In other work, several authors use experiments to study late bidding and ending rules. Houser and Wooders (2005) report the results of a controlled field experiment on late bidding behavior; however, they do not consider whether late bidding is an equilibrium or dominant strategy. Ariely et al. (2006) find through laboratory human-subject experiments in second-price online auctions in a private value (PV) environment that hard-close promotes late bidding whereas soft-close does not.

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They assume the number of customers to be constant. By using finite automata to model bidding strategies, Duffy and Unver (2008) develop a model of online auctions to understand how ending rules (i.e., hard- or soft-close) affect bidding behavior. They report results from simulations involving populations of artificial customers who use a genetic algorithm to update their strategies.

However, the results obtained in the papers cited above are derived from different models or frameworks. Yang and Kahng (2006) use the rate equation approach, Ariely et al. (2006) and Houser and Wooders (2005) use experiments, and Duffy and Unver (2008) use finite automata and simulation. Moreover, all studies mentioned above are for second-price auctions and consider only whether late bidding is equilibrium; that how much to bid is not considered, and the number of customers is either not considered or assumed constant.

The most related paper to ours is Ockenfels and Roth (2006), who model a second-price online auction conducted over time, where a nonzero probability exists for very late bids to be rejected. They make many assumptions including that the number of customers is predetermined and that all customers arrive at the beginning of the auction. They show that, for hard-close auctions, late bidding is one of multiple equilibria (another possibility is for all customers to bid early) under PV and is an equilibrium in an example under uncertain, dependent values (or common values; CVs); for soft-close auctions, late bidding is not an equilibrium.

To bring theory closer to practice, the present study considers both first- and second-price online auctions with hard- or soft-close ending rule under PV or CV (so there are eight specifications). We present a simple but general method: First, we divide the auction process into two stages. The first stage is the period before the end of the auction (called *the normal period*) where customers arrive one after another and determine when and how much to bid. The second stage is near the end of the auction (called *the ending period*). We use backward induction to derive equilibrium bidding strategies. We find that late bidding is dominant under CV, but only in first-price online auctions with hard-close endings under PV. For second-price online auctions it is dominant to report true value for all customers, immediately upon arrival under PV but near the end of the auction under CV, irrespective of ending rules. Finally, the time at which customers bid does not depend on the ending rule (i.e., hard- or soft-close) except for first-price online auctions under PV.

This paper is organized as follows: Section 2 describes the model. Sections 3 and 4 derive the equilibrium bidding strategies for first-price online auctions with a hard- or soft-close, respectively, under PV. In Section 5, we extend the study to other settings, including first-price online auctions under CV, and second-price online auctions with either hard- or soft-close under PV or CV. Finally, Section 6 concludes the paper.

2. The model

Consider a seller with a single indivisible object to sell through an online auction. Before the auction begins, the seller announces a starting bid q. Thus, the first bid should not be less than q. On the web, the current bid (denoted by b) is the highest bid made by previous customers, and changes each time a new bid is entered. Assume that the minimum bid increment is $\varepsilon > 0$; that is, a new bid can be accepted if and only if it is greater than or equal to $b + \varepsilon$, the current bid plus the increment.

Customers arrive one after another on the web; no assumption restricting the arrival process is required. In fact, we only care about the number of arrivals, which is denoted by $\xi + 1$, and ignore the order in which the customers arrive. We assume that all customers have valuations greater than $b + \varepsilon$; otherwise they can be ignored. Let the probability distribution of ξ be

$$p(n) = P\{\xi = n\}, \quad n \geqslant 0. \tag{1}$$

That the number of customers is $\xi + 1$ means that at least one customer participates in the auction. This is necessarily true when one customer is considering its bid, facing ξ customers. Otherwise discussing the bidding behavior is unnecessary. Thus, each customer faces competition from ξ other customers in the auction.

Each customer values the object at an amount of v, known only to herself. The seller and the other customers are uncertain about this valuation, and it appears to them to be a random variable. We consider two types of customer valuations: The first is PV which means it is common-knowledge among all customers and the seller that everyone views other customer valuations as independent draws from a common distribution function $F(x)$ over $[0, \infty]$ with $F(0) = 0$ and $F(\infty) = 1$ for which there exists a positive, continuously differentiable probability density function $f(x)$. That is, everyone agrees that the prior probability that v is less than x is given by $F(x)$, and everyone knows that everyone knows this, ad infinitum. The second type of valuation is CV which means all customer valuations are related; see Bajari and Hortacsu (2003) for details. Assume furthermore that all customers are risk neutral; in this case, their objective is simply to maximize their own expected profits.

The seller considers two alternative ending rules to sell the object: hard- and soft-close. Under hard-close, the auction lasts a time $[0, T]$ and closes exactly at T. Soft-close also imposes a predetermined ending time T but adds another predetermined time interval δ. A soft-close auction closes if and only if no new bid is placed during a time period with length of δ after the epoch $T - \delta$. That is, the auction closes at T only if no new bid occurs during $[T - \delta, T]$; If a new bid occurs at $T - \delta'$ for some $\delta' < \delta$, then the auction will be prolonged to $[T - \delta', T - \delta' + \delta]$ to see if another new bid occurs during this time interval. In contrast to the hard-close ending, the soft-close ending means the duration of the auction is random. In general, δ is far less than T. For example, T may be 7 days whereas δ may be only 10 min.

Armed with the above information regarding the auction, we now describe the dynamics of the online auction. When a customer arrives at the auction site, she observes the current bid b. If its valuation is greater than or equal to $b + \varepsilon$, then she can bid immediately, bid later, or bid near the end of the auction. Suppose she bids x when the current bid is b. Of course, $x \geqslant b + \varepsilon$ must be satisfied. Thus, the current bid becomes x and the customer becomes the *high bidder* (who holds the current bid). The high bidder at the end of the auction will win the object and is called the *winner*. In *a first-price online auction*, the winner pays her bid; whereas in *a second-price online auction*, the winner pays the second highest bid (This is consistent with those for sealed-bid auctions given in Klemperer (2000)).

Combining first- or second-price online auctions, PV or CV, and soft- or hard-close endings generates different games among customers, with each customer having to decide when and how much to bid as a function of her valuation of the object. Given that customers are symmetric, it is natural to focus on symmetric sequential equilibrium bidding strategies. At symmetric sequential equilibrium, each customer bids an amount that is some function of her own valuation (denoted by $B(\cdot)$) such that, given that everyone else determines their bids in the same way, no individual customer can do better by bidding differently. Assume that $B(\cdot)$ is increasing in valuation, as is well-known in the auction literature (e.g., Maskin and Riley 1987, p. 415, McAfee and McMillan 1987, Riley and Samuelson 1981, p. 383).

3. First-price online auction with hard-close under private value

The motivations to study first-price online auctions are as follows. First, a hard-close auction in eBay can be divided into