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Critical mass and willingness to pay for social networks

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ABSTRACT

Disagreement surrounds a formal definition of 'critical mass' and of the economic willingness to pay for membership in a social network. Our paper adapts work from percolation theory to analyze the structure of social networks, and draws an analogy for critical mass in social networks to the concept of phase changes in materials. We show how network growth can be actively managed, and define how to manage the willingness to pay for membership. We show, if achieving a critical mass of members in a social network is our objective, that prior to achieving critical mass, (1) the probability of accepting an invitation must vary inversely with individuals' breadth of contacts; and (2) the number of special interest groups of any size will decrease following a power law until immediately below critical mass. Targeted invitations enabled through sophisticated programs such as AdWords and IndexTools can help to actively maximize the probability of forming an acquaintance link. Our model defines a willingness to pay for network membership that is nearly zero below critical mass, and is an involved function above critical mass whose shape appears to be close to a logarithmic function. Our robust measure of the connectedness of members of a particular social network yields values that are consistent with the independently developed metrics of Odlyzko and Tilly [Odlyzko, A., and Tilly, B. A refutation of Metcalfe's Law and a better estimate for the value of networks and network interconnections, 2005 (downloaded from http://www.dtc.umn.edu/~odlyzko July 3, 2008)], and differ from eponymous 'laws' of Sarnoff, Metcalfe and Reed. There also appears to be plausible evidence in support of the market actually pricing networks at values close to Odlyzko and Tilly's estimates.

1. Social networks

Social networks are structures consisting of *members* (represented as nodes on a network graph) that share one or more specific types of *special interests*, such as values, visions, ideas, financial exchange, friends, kinship, dislike, conflict, trade, web links, and so forth (represented as links on a network graph). Social networking software and services facilitate the formation and management of online Internet member communities. This study builds a model of social networking that attempts to answer two questions:

(1) What is a network's '*critical mass*' of members, required for growth to become self-sustaining?
(2) How do you manage the social network structure so that the individual member's '*willingness to pay*' for membership in the network is sufficient to reach critical mass, and for self-sustaining growth?

O'Reilly (2005) initiated a widespread interest in Web based social networks (what he called Web 2.0) by showing how they had become central to the dot-com business models that had survived and thrived after 2001. Social networks such as MySpace and Facebook are web-based and offer various ways for users to interact. Web 2.0 platforms 'mash-up' complementary data flows and exploit social networks to generate new data (*eBay*, *Craigslist*, *Wikipedia*, and *del.icio.us* were given by O'Reilly as examples).

Social networks and Web 2.0 are arguably the most commercially significant World Wide Web platforms introduced in the past few years. Social network software is now a significant area of investment, and questions of 'critical mass' and 'willingness to pay' for access to particular networks are central to the investment in and operation of social network platforms.

Critical mass in social network research is a sociodynamic term used to describe the scale of a social system at which the system becomes self-sustaining and fuels further growth (Ball 2004), though often research has been hard pressed to move beyond 'perceived critical mass' as a researchable construct (e.g., see Van Slyke et al. 2007, Lou et al. 2000). In itself, this is problematic, since if the field has a difficult time articulating what exactly critical mass *is*, then it is unlikely that 'perceptions' will be consistent, accurate or useful, which may tend to undermine the credibility of conclusions from behavioral research on social networks. To consistently measure perceived critical mass, one first needs formal metrics of critical mass. An important goal of this research is to define a

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formal metric for critical mass in Internet based social networks, and to model and predict the behavioral transition that occurs at, before and after critical mass.

Investment and valuation in social network platforms has been hampered by a poor understanding of the structure of networks, and how these impact the potential return on investment in social networking platforms. It is widely asserted that non-linear cost-benefits exist in network investments, but several competing theories and rules-of-thumb compute widely differing values for costs and benefits.

Our analysis of ‘critical mass’ and ‘willingness to pay’ begins with a review of the supporting literature (Section 2). Assumptions, mathematical notation and key terms are listed in Tables 1–3 respectively. We review various models of network ‘willingness to pay’ that have been proposed over the past two decades in Section 3. A model of network growth based on well-established constructs from percolation theory is developed (Section 4). This model is used to predict growth, value and cost below and above the point of ‘critical mass’ (Sections 5). These results are then discussed in the context of managerial strategies, potential applications and future research (Section 6) with a final discussion of implications for crowd behavior and other anecdotal observations (Section 7).

2. Network models and critical mass

Social networks have frequently been addressed in the context of random, scale-free and small world networks (which are not necessarily mutually exclusive categorizations). In particular, Watts (Watts and Strogatz 1998, Watts 1999, Dodds and Watts 2004, Watts et al. 2005) and Barabási (Barabási and Albert 1999, Albert and Barabási 2002, Barabási and Oltvai 2004, Barabási et al. 2006) have played central roles in model development in social networks.

In general the models used to understand social networks have not exhibited a ‘critical transition, though anecdotal evidence (e.g., see examples throughout Shapiro and Varian, 1998) asserts the existence of a unique critical mass.’ This partly has to do with the objectives and particular metrics that have been the focus of the research; but it also reflects the fact that only a fraction of commonly invoked

Table 1
Key assumptions in the modeling and analysis.

Network mathematical structure	Connected acyclic graph (Bethe lattice)
Time	Time is compressed out of the analysis
Cycles	Chains of social relationships can be approximated with an acyclic graph
Social links	Number is fixed maximum number of relationships (links) for each member
Population	Network members (nodes) are drawn from an infinite population

Table 2
Mathematical notation.

Variable	Definition
p	The probability that an individual member of the network will form an acquaintance link to another member
p_c	Critical probability p_c at which a ‘phase change’ occurs and the ‘giant cluster’ appears
z	Coordination number of the Bethe Lattice; the maximum number of links that any member may create
g	Germanity, the probability that that any arbitrary member on the social network is connected to the giant cluster; g is non-zero only after phase change when the giant cluster appears
h	$1 - g$
n_s	The cluster number $n_s = p^s(1-p)^2$ as the probability of an arbitrary node being at the end of the cluster nearest the ‘origin’
s	Cluster size
u	Average cluster size
τ	Fisher exponent

Table 3
Key terms and their definitions.

Key term	Definition
Bethe lattice	A network model that is a connected acyclic graph and is used in percolation theory, introduced by Hans Bethe in 1935
Coordination number	A Bethe lattice construct that represents, the maximum number of direct links available to another node on the network
Critical mass	In the context of this model, the $p = p_c$ at which a phase change occurs and the giant cluster appears
Dunbar’s number	150, an upper limit in number of acquaintance links due to the limitations of human cognition; named after anthropologist Robin Dunbar
Germanity	The probability that that any arbitrary member on the social network is connected to the giant cluster; in this paper used as a measure of ‘willingness to pay’ and for ‘strength of network effect’
Giant cluster	A largest cluster in a network that has crossed over to critical behavior (Newman, 2001 termed this the ‘giant component’ which we have altered to ‘giant cluster’ in this paper, as this was felt to be more consistent with the general terminology used in social networks).
Lattice	Another term for a network, but usually connoting one built from repeating polygons. Lattice models appear often in percolation literature.
Measure of perceived connectedness	The logarithm of the average number (per member) of special interest clusters containing s members each – $\log(\bar{n}_s(p))$
Percolation theory	The study of probabilistic models that exhibit a ‘phase transition’
Universality principle	A concept in statistical mechanics. In a social network context, universality implies that the behavior of crowds (clusters) is a more universal characteristic than the particular associations that are made at an individual level. Individual behavior may vary considerably, and the background and preferences of individuals in a network may vary significantly, but the structure of the network will still be revealed in crowd behavior
Willingness to pay	The willingness to pay for network membership is assumed to exist once the giant cluster appears, and is proportional to the *germanity* g – the probability that that any arbitrary member on the social network is connected to the giant cluster