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New discrete-time robust H_2/H_∞ algorithm for vibration control of smart structures using linear matrix inequalities

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ABSTRACT

In real structural systems, such as a building structure or a mechanical system, due to inherent structural modeling approximations and errors, and changeable and unpredictable environmental loads, the structural response unavoidably involves uncertainties. These uncertainties can reduce the performance of a control algorithm significantly and possibly make it unstable. In this paper, based on the theories of the Bounded Real Lemma and the linear matrix inequalities (LMI), a novel discrete-time robust H_2/H_∞ control algorithm is presented which not only reduces the structural peak response caused by external dynamic forces but also is robust and stable in the presence of parametric uncertainties which is always the case in real-life structures. To facilitate practical implementation, the uncertainties of structural parameters are considered in the time domain as opposed to the frequency domain. Compared with traditional H_∞ control methods, the new control algorithm proposes a convenient design procedure to facilitate practical implementations of active control of complex and large structural systems through the use of a quadratic performance index and the LMI-based solution method. The effectiveness of the new discrete-time robust H_2/H_∞ adaptive control algorithm is demonstrated using a three-story frame with active bracing systems (ABS) and a ten-story frame with an active tuned mass damper (ATMD).

1. Introduction

Reducing the peak response quantities such as displacements and accelerations of structures subjected to external dynamic loads is of primary concern in design of large structures. The most recent design strategies focus on methods of structural vibration control (Adeli and Saleh, 1999; Adeli and Jiang, 2009; Adeli and Kim, 2009). These methods are divided into passive control such as the Tuned Mass Damper (TMD) (Gutierrez-Soto and Adeli, 2013a; Andersson et al., 2015), semi-active control (Fisco and Adeli, 2011a), active control (Kim and Adeli, 2005d; Gutierrez-Soto and Adeli, 2013b), and hybrid control (Kim and Adeli, 2005b, 2005c; Fisco and Adeli, 2011b) method. Compared to the passive control system, an active control system has advantages of adaptability and performance. Moreover, semi-active and hybrid control strategies which are more practical in terms of implementation are always based on active control algorithms (El-Khoury and Adeli, 2013).

Over the past few decades many active control algorithms have been developed such as the linear quadratic regulator (LQR) (Stavroulakis et al., 2006), linear quadratic Gaussian (LQG) (Wu and Yang, 2000), sliding mode control (SMC) (Alli and Yakut, 2005; Pai, 2010; Wang and Adeli, 2012, 2015a, 2015b), H_∞ control (Yang et al., 1996), proportional–integral–derivative (PID) control (Kang et al., 2009), model predictive control (Wang et al., 2015), parallel control (Li et al., 2014), and optimal control algorithm (Adeli and Saleh, 1997; Saleh and Adeli, 1997, 1998a, 1998b, Li et al., 2015). Adeli and Saleh (1998, 1999) present an integrated control and optimization strategy for design of both civil structures and control system. For solution of the integrated control and optimization Saleh and Adeli (1994) present parallel algorithms on high-performance parallel machines (Adeli and Kamal, 1993) and supercomputers (Adeli and Soegiarso, 1999). A review of recent advances on vibration control of structures under dynamic loading is presented by Khoury and Adeli (2013)

In real structures, due to inherent structural modeling approximations and errors, and changeable and unpredictable environmental loads, the structural system response unavoidably involves uncertainties. These uncertainties can reduce the performance of a control algorithm and possibly make it unstable. In the presence of structural parameters uncertainties traditional control methods do not provide the stability and robustness needed for effective reduction of the structural response under unknowable and varying external dynamic loading conditions. They can affect the structure adversely when the frequency of external

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disturbance is close to the natural frequency of the structure. Kim and Adeli (2004a,b) developed a hybrid feedback-least mean square (LMS) algorithm and an improved wavelet-hybrid feedback-LMS algorithm to suppress both steady and transient vibrations effectively. They demonstrated their effectiveness for significant vibration reduction in both irregular highrise building (Kim and Adeli, 2005a) and bridge structures (Kim and Adeli, 2005d).

Based on the merits of neural networks and fuzzy logic methods for system identification (Boutalis et al., 2013; Rigatos, 2013), Jiang and Adeli (2008a, 2008b) present a dynamic fuzzy wavelet neuroemulator to predict the structural response, and then obtain the optimal control forces using the genetic algorithm. Amini and Zabihi-Samani (2014) present a time varying wavelet-based pole assignment (WPA) method to control the seismic response of a building structure. However, the uncertainties of the structural parameters, such as stiffness, damping, and mass, are not taken into account in any of these methods.

A key issue in practical implementation of the vibration control technology is the robustness of the control algorithm in the presence of uncertainty which is the subject of this paper. H_∞ control is a popular control algorithm in the area of robust control that requires the solution of the Riccati equation (Saleh and Adeli, 1997; Zhou and Doyle, 1998) which is time consuming in terms of computing. A few researchers have used the H_∞ control for vibration control of structures based on the continuous time system. Yang et al. (1996) show that the H_∞ control method is effective for reducing the seismic response of building structures but do not consider uncertainties of structural parameters in the formulation. Calise and Sweriduk (1998) describe an H_∞ control method based on the frequency domain but the uncertainties of structural parameters cannot be described easily in the frequency domain. Wang et al. (2004) present an H_∞ controller taking into account the norm-bounded uncertainties of structural parameters through solving a Riccati equation and choosing a set of flexible scalars. Simulation results show the effectiveness of the algorithm with a perturbation of 10% for mass, stiffness and damping coefficients. However, solving the resulting Riccati equation with excessive flexible scalars can be problematic and the performance of the control algorithm with a relatively high perturbation can deteriorate. The aforementioned H_∞ control methods all require the solution of the Riccati equation.

An effective approach for solution of the Riccati equation is application of the linear matrix inequalities (LMI) (Gahinet and Apkarian, 1994). Du et al. (2004), Wu et al. (2006), and Du et al. (2011) used an LMI-based solution approach for design of H_∞ controllers. Chang (2005) designed a mixed H_2/H_∞ control algorithm based on the frequency domain and applied it to the gap control of the electric discharge machines. Guimaraes et al. (2007) proposed an immune-based H_2/H_∞ controller. Yang et al. (2014) used a mixed H_2/H_∞ control algorithm to control the temperature of the four-zone split inverter air conditioners. These methods, however, have shortcomings because either they do not consider the structural parameter uncertainties in their formulation or deal with them in the frequency domain which makes their application to real structures difficult.

Since real-life control systems are modeled as a discrete-time system, an operational discrete-time robust control algorithm is of paramount importance for practical implementation of vibration control of structures with uncertainties. In this paper, a novel discrete-time robust H_2/H_∞ control algorithm is presented for vibration control of structures subjected to dynamic loading such as strong ground motions taking into account the uncertainties in modeling the structure based on the theories of the Bounded Real Lemma (Zhou and Doyle, 1998) and using the LMI approach (Boyd et al., 1994). To facilitate practical implementation, the uncertainties of structural parameters are considered in the time domain as opposed to the frequency domain. The H_∞ approach is employed to achieve stability. To increase the effectiveness of the control algorithm it is integrated with an H_2 algorithm where a quadratic performance index is used to evaluate and compare the performance of the control system. The effectiveness of the new discrete-time robust H_2/H_∞ control algorithm is demonstrated using a three-story frame with active bracing systems (ABS) and a ten-story frame with an active tuned mass damper (ATMD).

2. Modeling of uncertainty in the control equations

The dynamic equation of motion for an n-Degree-of-Freedom (DOF) structure subjected to one-dimensional ground acceleration $w(t)$ and active control forces is described as follows:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = -\mathbf{M}\bar{\mathbf{I}}w(t) + \mathbf{B}_s\mathbf{U}(t) \tag{1}$$

where $\mathbf{X}$ is the column vector of displacements relative to the ground, $\mathbf{M}$=diag[m_1, m_2, …, m_n] is the diagonal mass matrix, $\mathbf{K}$ and $\mathbf{C}$ are $n \times n$ stiffness and damping matrices, respectively, $\mathbf{B}_s$ is the $n \times r$ control device location matrix, r is the number of active control devices, $\mathbf{U}(t)$ is the r-dimensional vector of control forces, and $\bar{\mathbf{I}}$ is an $n \times 1$ vector whose values are all equal to one.

In this research uncertainties of the structural parameters are described through perturbations of the parameters. As such, the dynamic equation of the structure with uncertainties can be described in the following manner:

$$\begin{aligned}(\mathbf{M} + \boldsymbol{\Delta}_\mathrm{M})\ddot{\mathbf{X}} + (\mathbf{C} + \boldsymbol{\Delta}_\mathrm{C})\dot{\mathbf{X}} + (\mathbf{K} + \boldsymbol{\Delta}_\mathrm{K})\mathbf{X} \\ = -(\mathbf{M} + \boldsymbol{\Delta}_\mathrm{M})\bar{\mathbf{I}}w(t) + (\mathbf{B}_s + \boldsymbol{\Delta}_{\mathrm{B_S}})\mathbf{U}(t)\end{aligned} \tag{2}$$

where $\boldsymbol{\Delta}_\mathrm{M}$, $\boldsymbol{\Delta}_\mathrm{K}$, $\boldsymbol{\Delta}_\mathrm{C}$and $\boldsymbol{\Delta}_{\mathrm{B_S}}$ are corresponding perturbations prescribed using a preselected scalar $0 < \delta_i < 1$ so that $0 < \Delta_{Mi}/M_i \le \delta_i < 1$ which can guarantee $\mathbf{M} + \boldsymbol{\Delta}_\mathrm{M}$ is non-singular. Consequently, the uncertainty $\boldsymbol{\Delta}_\mathrm{M}$ satisfies the following bound where $\boldsymbol{\delta}$ is an $n \times n$ diagonal matrix with terms δ_i in the diagonal (δ_i is chosen according to amplitude of uncertainty $\boldsymbol{\Delta}_\mathrm{M}$):

$$\|\boldsymbol{\Delta}_M\mathbf{M}^{-1}\| \le \|\boldsymbol{\delta}\| < 1 \tag{3}$$

To avoid the problematic inversion of $\mathbf{M} + \boldsymbol{\Delta}_\mathrm{M}$, consider two different scalar matrices $\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$, and assume the following relationship holds: $(\mathbf{I} + \boldsymbol{\delta})(\mathbf{I} + \boldsymbol{\delta}') = \mathbf{I}$. Then, Eq. (2) can be rearranged as follows:

$$\begin{aligned}\ddot{\mathbf{X}} + (\mathbf{I} + \boldsymbol{\delta}')\mathbf{M}^{-1}(\mathbf{C} + \boldsymbol{\Delta}_\mathrm{C})\dot{\mathbf{X}} + (\mathbf{I} + \boldsymbol{\delta}')\mathbf{M}^{-1}(\mathbf{K} + \boldsymbol{\Delta}_\mathrm{K})\mathbf{X} \\ = -\bar{\mathbf{I}}w(t) + (\mathbf{I} + \boldsymbol{\delta}')\mathbf{M}^{-1}(\mathbf{B}_s + \boldsymbol{\Delta}_{\mathrm{B_S}})\mathbf{U}(t)\end{aligned} \tag{4}$$

Following the state space representation, Eq. (4) can be expressed in the following form:

$$\dot{\mathbf{Z}}(t) = (\mathbf{A} + \Delta\mathbf{A})\mathbf{Z}(t) + (\mathbf{B} + \Delta\mathbf{B})\mathbf{U}(t) + \mathbf{H}w(t) \tag{5}$$

where

$$\Delta\mathbf{A} = \begin{bmatrix} 0 & 0 \\ -\mathbf{M}^{-1}\boldsymbol{\delta}_K & -\mathbf{M}^{-1}\boldsymbol{\delta}_C \end{bmatrix}, \quad \Delta\mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\boldsymbol{\delta}_{B_S} \end{bmatrix}, \quad \mathbf{Z}(t) = \begin{bmatrix} \dot{\mathbf{X}} \\ \mathbf{X} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{B}_s \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 0 \\ -\{1\} \end{bmatrix},$$

$$\boldsymbol{\delta}_K = (\mathbf{I} + \boldsymbol{\delta}')\boldsymbol{\Delta}_K + \boldsymbol{\delta}'\mathbf{K}, \;\; \boldsymbol{\delta}_C = (\mathbf{I} + \boldsymbol{\delta}')\boldsymbol{\Delta}_C + \boldsymbol{\delta}'\mathbf{C}, \;\; \boldsymbol{\delta}_{B_S} = (\mathbf{I} + \boldsymbol{\delta}')\boldsymbol{\Delta}_{B_S} + \boldsymbol{\delta}'B_s$$

Because the control system expressed by Eq. (5) is linear and time-invariant, the following analytical solution is obtained (Zhou and Doyle, 1998):

$$\begin{aligned}\mathbf{Z}(t) = e^{(\mathbf{A}+\Delta\mathbf{A})(t-t_0)}\mathbf{Z}(t_0) + \int_{t_0}^{t} e^{(\mathbf{A}+\Delta\mathbf{A})(t-\tau)}(\mathbf{B} + \Delta\mathbf{B})\mathbf{u}(\tau)d\tau \\ + \int_{t_0}^{t} e^{(\mathbf{A}+\Delta\mathbf{A})(t-\tau)}\mathbf{H}w(\tau)d\tau\end{aligned} \tag{6}$$