



# Combining CSP and MPC for the operational control of water networks



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## ARTICLE INFO

### Article history:

Received 4 March 2015

Received in revised form

6 December 2015

Accepted 7 December 2015

Available online 30 December 2015

### Keywords:

MPC

CSP

Epanet

Water networks

## ABSTRACT

This paper presents a control scheme which uses a combination of linear Model Predictive Control (MPC) and a Constraint Satisfaction Problem (CSP) to solve the non-linear operational optimal control of Drinking Water Networks (DWNs). The methodology has been divided into two functional layers: first, a CSP algorithm is used to transfer non-linear DWNs pressure equations into linear constraints on flows and tank volumes, which can enclose the feasible solution set of the hydraulic non-linear problem during the optimization process. Then, a linear MPC with tightened constraints produced in the CSP layer is solved to generate control strategies which optimize the control objectives. The proposed approach is simulated using Epanet to represent the real DWNs. Non-linear MPC is used for validation. To illustrate the performance of the proposed approach, a case study based on the Richmond water network is used and a realistic example, D-Town benchmark network, is added as a supplementary case study.

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## 1. Introduction

Water is always a critical resource for supporting human activities and ecosystem conservation. Recently, the population and users' requirements are increasing while water resources are limited. This situation indicates the need for an optimal operation of water distribution networks, especially during shortage events as discussed in Miao et al. (2014) and Soltanjalili et al. (2013). Management of Drinking Water Networks (DWNs) involves objectives such as minimizing operational cost of pumps, which represents a significant fraction of the total expenditure of a water utility, as discussed in López-Ibañez et al. (2008), or minimizing risks of service failure (as explained in Kurek and Ostfeld, 2014).

The optimization problems associated to the operational control of DWNs are complex because of their large-scale, multiple-input, multiple-output nature, as well as the various sources of additive and, possibly, parametric uncertainty in DWNs. Additionally, DWNs models include both deterministic and stochastic components and involve linear (flow model) as well as non-linear (pressure model) equations. The use of non-linear models in DWNs is essential for the operational control which involves manipulating not only flows but also pressures.

Non-linear optimization refers to optimization problems where the objective or constraint functions are nonlinear, and possibly non-convex. No universally applicable methods exist for solving a

non-linear optimization problem when it is non-convex. Even simple-looking problems with a small number of variables can be extremely challenging, while problems with important number of variables can be intractable. Non-linear optimization may be addressed with several different approaches; each of which involving some compromise. Local optimization methods can be fast and can also handle large-scale problems although they do not guarantee finding the global optimum. Alternatively, global optimization is limited to be used in small problems (networks), where computational time is not critical, because usually the global solution search is time consuming, as discussed in Boyd and Vandenberghe (2004).

Early optimization approaches for DWNs typically rely on a substantially simplified network hydraulic model (by dropping all nonlinearities, for instance) as described in Coulbeck et al. (1988), Diba et al. (1995), Sun et al. (1995) and Papageorgiou (1983), which is often unacceptable in practice. Other authors employ discrete dynamic programming as presented in Can and Houck (1984), Carpentier and Cohen (1993), Cembrowicz (1990), Murray and Yakowitz (1979), Orr et al. (1990) and Zessler and Shamir (1989), which is mathematically sound but only applicable to small networks unless specific properties can be exploited to increase efficiency.

Model Predictive Control (MPC) is a well-established class of advanced control methods for complex large scale systems, as explained in Rawlings and Mayne (2009) and Mayne et al. (2000). In Ocampo-Martínez et al. (2013) and Fiorelli et al. (2013), MPC has been successfully applied to control and optimize linear flow

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model of DWNs. When the pressure model is considered, the non-linear functions involved will increase the computational burden of MPC especially when the size of the network increases. Besides, convergence to the global minimum cannot be easily guaranteed using non-linear MPC if non-linear programming algorithms are used. As described in [Boyd and Vandenberghe \(2004\)](#), for a non-convex problem, an approximate, but convex formulation is needed. By solving the approximate problem, which can be done easily and without an initial guess, the exact solution to the approximate convex problem is obtained. Many methods for global optimization require a cheaply computable lower bound on the optimal value of the non-convex problem. In the relaxed problem, each non-convex constraint is replaced with a looser, but convex constraint. In [Mayne et al. \(2011\)](#), a similar approach based on tube-based MPC is proposed. In this case, the way of circumventing the complexity problem is based on replacing the non-linear MPC by an approximation about a nominal trajectory. Trajectories are bounded by a level set of a value function that varies in a complex way with, the state and time.

This paper mainly provides a methodology for solving large scale complex non-linear DWNs problem using a convex approximation of the problem. The solution is compared to that of a nonlinear MPC implementation, obtained with a tool named PLIO ([Cembrano et al., 2011](#)). Simulation results are compared using the Richmond case study introduced in [van Zyl et al. \(2004\)](#). Finally, the D-Town benchmark network, which is much more realistic as presented in [Price and Ostfeld \(2014\)](#) and [Iglesias-Rey et al. \(2014\)](#), is used as a supplementary case study for validation.

The aim of the proposed approach is to avoid the non-linear optimization problem of DWNs by the combined use of linear MPC and CSP while maintaining optimality and also feasibility with the tightened linear constraints provided by the CSP in [Streif et al. \(2014\)](#). To assess the proposed approach, the real hydraulic behavior of the DWNs is simulated by means of Epanet ([Rossman, 2000](#)), which simulates DWNs using the input optimal solution provided by MPC. As shown in [Fig. 1](#), the whole controlling methodology works in a two-layer structure as initially proposed in [Sun et al. \(2014a\)](#): CSP is the first step of this methodology and it constitutes the upper layer used for converting the non-linear hydraulic pressure constraints into the linear MPC constraints. MPC is the lower layer producing optimal set-points for controlling actuators (pumps and valves), according to the defined objective functions including minimizing operational costs of pumps, risks and safety goals.

The remainder of the paper is organized as follows: The control-oriented modelling methodology considering both flow and pressure dynamics is presented in [Section 2](#). Then, in [Section 3](#), the operational control problem is introduced in the context of non-linear MPC. In [Section 4](#), the definition of CSP and also the proposed CSP-MPC control scheme are explained in detail. [Section 5](#) summarizes the results and validations using the Richmond case study.

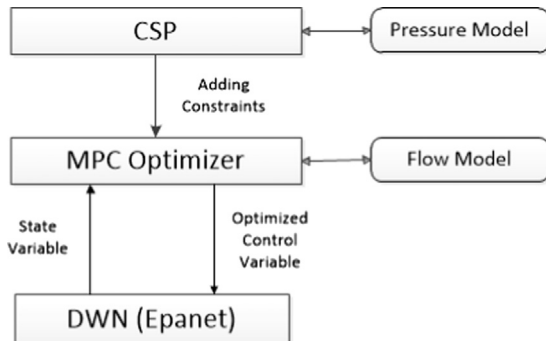


Fig. 1. The multi-layer control scheme.

[Section 6](#) provides a supplementary application based on a more complex example, a benchmark network called D-Town. Finally, [Section 7](#) contains the conclusions and future research plans.

## 2. Control-oriented modelling methodology

Drinking Water Networks (DWNs) generally contain tanks, which store the drinking water at appropriate head level (elevation and pressure) to supply demand, a network of pipes and a number of demands. Valves and/or pumping stations are the elements that allow to manipulate the water flow according to a specific policy and to supply water requested by the network users at appropriate service pressures.

The DWNs can be considered as composed of a set of constitutive elements, which are presented below including first the flow model and then the pressure model.

### 2.1. Flow model

#### 2.1.1. Reservoirs and tanks

Water reservoirs and tanks play an important role in DWNs since they enable demand management, ensure water supply (e.g., in case of unexpected demand changes or in case of emergencies) and allow for the modulation of pump flow rate as discussed in [Batchabani and Fuamba \(2014\)](#) and [Lee et al. \(2013\)](#). Moreover, they provide the entire network with the water storage capacity. The mass balance expression of these storage elements relates the stored volume  $V$ , the manipulated inflows  $q_{in}^i$  and outflows  $q_{out}^h$  (including the demand flows as outflows). The  $i$ th storage element can be described by the discrete-time difference equation

$$V_i(k+1) = V_i(k) + \Delta t \left( \sum_j q_{in}^j(k) - \sum_h q_{out}^h(k) \right), \quad (1)$$

where  $\Delta t$  is the sampling time and  $k$  denotes the discrete-time instant. The physical constraint related to the admissible range of water levels in the  $i$ th storage element is expressed as

$$\underline{V}_i \leq V_i(k) \leq \bar{V}_i, \quad \text{for all } k, \quad (2)$$

where  $\underline{V}_i$  and  $\bar{V}_i$  denote the minimum and the maximum admissible storage capacity, respectively. Although  $\underline{V}_i$  might correspond to an empty storage element, in practice this value is normally set as nonzero in order to maintain an emergency stored volume for extreme circumstances.

For simplicity purposes, the dynamic behavior of these elements is described as function of volume. However, in most cases, the measured variable is the storage water level (by using level sensors), which implies the computation of the water volume taking into account the tank geometry.

#### 2.1.2. Actuators

Two types of control actuators are considered: valves and pumps (more precisely, complex pumping stations). In the flow model, valves and pumps are simplified and considered as similar control elements, and their flows are taken as the manipulated variables in the MPC problem, denoted as  $q_{ui}$ . Both pumps and valves have lower and upper physical limits, which are taken into account as system constraints. As in (2), they are expressed as

$$q_{u,i} \leq q_{ui}(k) \leq \bar{q}_{ui}, \quad \text{for all } k, \quad (3)$$

where  $q_{u,i}$  and  $\bar{q}_{ui}$  denote the minimum and the maximum flow capacity, respectively.

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