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New distance and similarity measures on hesitant fuzzy sets and their applications in multiple criteria decision making ☆

Deqing Li [a,b], Wenyi Zeng [b,*], Junhong Li [b,c]

[a] Department of Mathematics, Shijiazhuang Mechanical Engineering College, Shijiazhuang 050003, PR China
[b] College of Information Science and Technology, Beijing Normal University, Beijing 100875, PR China
[c] College of Mathematics and Information Science, Hebei Normal University, Shijiazhuang 050024, PR China

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ABSTRACT

The distance measures of hesitant fuzzy elements (HFEs) $h_1(x)$ and $h_2(x)$ introduced in the literature only cover the divergence of the values, but fail to consider the difference between the numbers of value of $h_1(x)$ and $h_2(x)$. However, the main characteristic of HFE is that it can describe the hesitant situations flexibly. Such a hesitation is depicted by the number of values of HFE being greater than one. Hence, it is very necessary to take into account both the difference of the values and that of the numbers when we study the difference between the HFEs. In this paper, we introduce the concept of hesitance degree of hesitant fuzzy element which describes the decision maker's hesitance in decision making process. Several novel distance and similarity measures between hesitant fuzzy sets (HFSs) are developed, in which both the values and the numbers of values of HFE are taken into account. The properties of the distance measures are discussed. Finally, we apply our proposed distance measures in multiple criteria decision making to illustrate their validity and applicability.

1. Introduction

The theory of fuzzy sets proposed by Zadeh (1965) has achieved a great success in various fields. The extensions of fuzzy set have been developed by some researchers, including the interval-valued fuzzy set introduced by Zadeh (1975), intuitionistic fuzzy set pioneered by Atanassov (1986), interval-valued intuitionistic fuzzy set introduced by Atanassov and Gargov (1989), type-2 fuzzy set pioneered by Dubois and Prade (1980) and fuzzy multiset introduced by Yager (1986). In the practical decision making process, it is difficult to establish the degree of membership of fuzzy set because of the time pressure, lack of knowledge or data and some other reasons. To deal with these cases, Torra (2010) and Torra and Narukawa (2009) introduced the concept of hesitant fuzzy set which permitted the membership degree having a set of possible values. It can reflect the human's hesitancy more objectively than the other classical extensions of fuzzy set, hence it is a very useful tool to deal with uncertainty, and has become a popular topic in multiple attribute decision making in hesitant fuzzy environment. Some researchers have paid attention on this topic and have established some aggregation operators based on hesitant fuzzy set. For example, Xia and Xu (2011) developed a series of aggregation operators for hesitant fuzzy information, and further discussed the correlations among the aggregation operators. Chen et al. (2013b) investigated correlation coefficient of hesitant fuzzy sets, Wei (2012) investigated hesitant fuzzy prioritized operators, Zhang and Wei (2013) investigated the extension of VIKOR method based on hesitant fuzzy set, Zhang (2013) investigated hesitant fuzzy power aggregation operators, Zhu et al. (2012) investigated hesitant fuzzy geometric Bonferroni means, Yu et al. (2012) investigated generalized hesitant fuzzy Bonferroni mean, Chen et al. (2013a) developed interval-valued hesitant fuzzy sets, Rodríguez et al. (2012) investigated hesitant fuzzy linguistic term sets for decision making. Onar et al. (2014) utilized hesitant fuzzy TOPSIS method to select the best strategy in strategic decisions.

Distance and similarity measure are two important topics in the fuzzy set theory and have been extensively applied in some fields such as decision making, pattern recognition, machine learning and market prediction and so on. The similarity measure indicates the similar degree of two fuzzy sets. Wang (1983) first put forward the concept of fuzzy sets' similarity measure and gave a computation formula. Since then, similarity measure of fuzzy sets has attracted some researchers' interest and has been investigated further. For example, Zwick et al. (1987) and Pappis and Karacapilidis (1993) investigated geometric distance and Hausdorff metrics and gave a comparative analysis of similarity measures of fuzzy sets, respectively. Wang (1997) proposed two new similarity measures between fuzzy sets and between elements. Zeng and Li (2006) investigated the

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* Corresponding author. Tel.: +86 10 58805665; fax: +86 10 58808313.
E-mail address: zengwy@bnu.edu.cn (W. Zeng).

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relationship among the inclusion measure, the similarity measure, and the fuzziness of fuzzy sets. Szmidt and Kacprzyk (2000) gave a new distance measure between two intuitionistic fuzzy sets by taking into account all three parameters of intuitionistic fuzzy set. Narukawa and Torra (2006) introduced a weighted distance measure for intuitionistic fuzzy sets based on the Choquet integral with respect to the non-monotonic fuzzy measure. Li et al. (2012) investigated the relationship between similarity measure and entropy of intuitionistic fuzzy sets.

Aimed at these two important numerical indexes in the fuzzy set theory, some researchers extended these concepts to the hesitant fuzzy set theory and investigated their related topics from different points of view. For example, Xu and Xia (2011a,b) investigated the distance, similarity measure and correlation measure of hesitant fuzzy sets and hesitant fuzzy information, Peng et al. (2013) proposed the generalized hesitant fuzzy synergetic weighted distance measure and applied it to multiple criteria decision making, Rodríguez et al. (2014) investigated hesitant fuzzy sets and its future directions. Aimed at these existing distance measures for hesitant fuzzy sets and hesitant fuzzy elements, the difference between the membership values is taken into account, but the difference of hesitance degree between the hesitant fuzzy elements is not considered. In fact, hesitancy is an important and essential characteristic of hesitant fuzzy sets. In this paper, we propose some new distance and similarity measures by taking into account the hesitance of the hesitant fuzzy sets and investigate their application.

The organization of our work is as follows. In Section 2, some basic notions of hesitant fuzzy sets are reviewed. In Section 3, we introduce the axiomatic definitions of distance and similarity measure of hesitant fuzzy sets, investigate the relationship between distance and similarity measure of hesitant fuzzy sets in detail, and put forward some new formulas to calculate the distance and similarity measures of hesitant fuzzy sets. In Section 4, we apply our proposed distance measures in multiple criteria decision making. The conclusion is given in the last section.

2. Preliminaries

Throughout this paper, we use $X=\{x_1,x_2,\dots,x_n\}$ to denote the discourse set, HFS and HFE stand for hesitant fuzzy set and hesitant fuzzy element, respectively, h stands for a HFS and $h(x)$ stands for a HFE, H stands for the set of all hesitant fuzzy sets in X.

Definition 1. (Torra, 2010) Given a fixed set X, then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1].

For conveniences, the HFS is often expressed simply by mathematical symbol in Wei (2012)

$$E=(\langle x,h_E(x)\rangle | x\in X)$$

where $h_E(x)$ is a set of some values in [0, 1], denoting the possible membership degree of the element $x\in X$ to the set E. $h(x)=h_E(x)$ is called a hesitant fuzzy element (HFE).

Definition 2. (Xu and Xia, 2011a) Let h_1 and h_2 be two hesitant fuzzy sets on $X=\{x_1,x_2,\dots,x_n\}$, then the distance measure between h_1 and h_2 is defined as $d(h_1,h_2)$, which satisfies the following properties:

(D1) $0\le d(h_1,h_2)\le 1$;
(D2) $d(h_1,h_2)=0$ if and only if $h_1=h_2$;
(D3) $d(h_1,h_2)=d(h_2,h_1)$.

Definition 3. (Xu and Xia, 2011a) Let h_1 and h_2 be two hesitant fuzzy sets on $X=\{x_1,x_2,\dots,x_n\}$, then the similarity measure between h_1 and h_2 is defined as $s(h_1,h_2)$, which satisfies the following properties:

(S1) $0\le s(h_1,h_2)\le 1$;
(S2) $s(h_1,h_2)=1$ if and only if $h_1=h_2$;
(S3) $s(h_1,h_2)=s(h_2,h_1)$.

Therefore, we have the following property.

Property 1. If d is the distance measure between hesitant fuzzy sets h_1 and h_2, then $s(h_1,h_2)=1-d(h_1,h_2)$ is the similarity measure between hesitant fuzzy sets h_1 and h_2.

Property 2. If s is the similarity measure between hesitant fuzzy sets h_1 and h_2, then $d(h_1,h_2)=1-s(h_1,h_2)$ is the distance measure between hesitant fuzzy sets h_1 and h_2.

To establish an order between HFEs, Xia and Xu (2011) introduced a comparison law by defining a score function. Farhadinia (2013) presented some counterexamples to show that the score function proposed by Xia and Xu was not able to discriminate some HFEs although they are apparently different, and introduced a new score function for HFEs as follows:

Definition 4. (Farhadinia, 2013) Let $h(x)=\bigcup_{\gamma\in h(x)}\{\gamma\}=\{\gamma_j\}_j^{l(h(x))}$ be a HFE, the score function S of the hesitant fuzzy element $h(x)$ is defined by

$$S(h(x))=\frac{\sum_{j=1}^{l(h(x))}\delta(j)\gamma_j}{\sum_{j=1}^{l(h(x))}\delta(j)}$$

where $\{\delta(j)\}_j^{l(h(x))}$ is a positive-valued monotonic increasing sequence of index j.

For two hesitant fuzzy elements $h_1(x)$ and $h_2(x)$, if $S(h_1(x))>S(h_2(x))$, then $h_1(x)>h_2(x)$; if $S(h_1(x))=S(h_2(x))$, then $h_1(x)=h_2(x)$.

It is noted that the number of elements in different hesitant fuzzy elements may be different, thus Rodríguez et al. (2014) have pointed out that the score function presented by Farhadinia (2013) was not able to distinguish some HFEs. Throughout this paper, we denote $l(h(x))$ as the number of elements in $h(x)$, and arrange the elements in it in descending order. In most cases, for two HFSs h_1 and h_2, $l(h_1(x))\neq l(h_2(x))$. To operate correctly, Xu and Xia (2011a) gave the following regulation: the shorter one is extended by adding the minimum value, the maximum value, or any value in it until it has the same length with the longer one. The selection of this value mainly depends on the decision makers' risk preferences. Optimists anticipate desirable outcomes and may add the maximum value, while pessimists expect unfavorable outcomes and may add the minimum value. Although the results may be different if we extend the shorter one by adding different values, it is reasonable because the decision makers' risk preferences can directly influence the final decision (Yu et al., 2012). In this paper, we extend the shorter one by adding the minimum value.

Moreover, for the distance measure of hesitant fuzzy elements, Xia et al. (2013) pointed out that $d(h_1(x),h_2(x))$ should satisfy the property in the following.

(D4) For three hesitant fuzzy elements $h_1(x)$, $h_2(x)$ and $h_3(x)$, which have the same length l and $h_k(x)=\{h_k^1(x),h_k^2(x),\dots,h_k^l(x)\}$, $k=1,2,3$, if $h_1^i(x)\le h_2^i(x)\le h_3^i(x)$, $i=1,2,\dots,l$, then

$$d(h_1(x),h_2(x))\le d(h_1(x),h_3(x)),\quad d(h_2(x),h_3(x))\le d(h_1(x),h_3(x))$$

Xu and Xia (2011a) proposed the hesitant normalized Hamming distance, Euclidean distance and generalized hesitant normalized