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A quantum-inspired gravitational search algorithm for binary encoded optimization problems

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ABSTRACT

In this paper, a novel population based metaheuristic search algorithm by combination of gravitational search algorithm (GSA) and quantum computing (QC), called a Binary Quantum-Inspired Gravitational Search Algorithm (BQIGSA), is proposed. BQIGSA uses the principles of QC such as quantum bit, superposition and a modified rotation Q-gates strategy together with the main structure of GSA to present a robust optimization tool to solve binary encoded problems. To evaluate the effectiveness of the BQIGSA several experiments are carried out on the combinatorial 0–1 knapsack problems, Max-ones and Royal-Road functions. The results obtained are compared with those of other algorithms including Binary Gravitational Search Algorithm (BGSA), Conventional Genetic Algorithm (CGA), binary particle swarm optimization (BPSO), a modified version of BPSO (MBPSO), a new version of binary differential evolution (NBDE), a quantum-inspired particle swarm optimization (QIPSO), and three well-known quantum-inspired evolutionary algorithms (QIEAs). The comparison reveals that the BQIGSA has merit in the field of optimization.

1. Introduction

The word "quantum computing (QC)" refers to creating a new generation of computers, called quantum mechanical computers, which work with a focus on the role of quantum mechanics in the theory of computation. In other words, a quantum computer is a device for computation that makes direct use of quantum mechanical phenomena to perform operations on data. The quantum computers were proposed in the early 1980s (Benioff, 1980; Feynman, 1982) and formalized in the late 1980s and 1990s (Deutsch, 1985; Shor, 1994; Grover, 1996). The basic principle behind the quantum computation is that the quantum properties can be used to represent data and execute operations on these data (Nielsen and Chuang, 2006). The development of polynomial time Shor's (1994) factoring algorithm and Grover's (1996) algorithm for quick search in unsorted database showed that the QC paradigm is more powerful than classical computing.

Evolutionary algorithms (EAs) as principle of evolutionary computing (EC) techniques refer to a group of nature-inspired algorithms for performing very complex search and optimization. EAs are metaheuristic global search methods and optimization algorithms modeled from natural genetic principles such as natural selection. The basic idea of natural selection is "Select the best, discard the rest". It means that better individuals get higher chance to survive. The important methods in the field of EAs are genetic algorithms (GA) proposed by Holland (1975), evolutionary programming (EP) proposed by Fogel et al. (1966), and evolutionary strategies (ES) proposed by Rechenberg (1973) and Schwefel (1975).

Swarm intelligence (SI) refers to a newly developed group of population-based algorithms for multi-agent search and optimization. SI studies the collective behavior of systems made up of a population of simple agents interacting locally with each other and with their environment. In the SI systems the agents follow very simple rules, although there is no centralized control structure dictating how individual agents should behave. Here, social interactions (locally shared knowledge) provide the basis for unguided problem solving. In recent years, the swarm intelligence paradigm has received tremendous attention in research, mainly as Ant Colony Optimization (ACO) and Particle Swarm Optimization (PSO). ACO is the first family of SI-based search algorithms which was proposed by Dorigo et al. (1991), where the behavior of ants is modeled in finding the shortest path from nest to food source. PSO, which was proposed by Kennedy and Eberhart (1995), mimics the flocking behavior of birds and fish.

Research on integrating EC and QC has been started since late 1990s. The existing algorithms in this field are classified by Zhang (2011) in three categories as follows:

- Evolutionary-designed quantum algorithms (EDQAs) which try to automate the synthesis of new quantum algorithms using EAs.

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- Quantum evolutionary algorithms (QEAs) which focus on implementation of EC algorithms in a quantum computation environment.
- Quantum-inspired evolutionary algorithms (QIEAs) which concentrate on generating new EC algorithms using some concepts and principles of QC.

Gravitational search algorithm (GSA) is one of the latest swarm optimization algorithms, which introduced by Rashedi et al. (2009) based on the metaphor of gravitational interaction between objects. Previous works have revealed the effectiveness and efficiency of the GSA as a global optimizer in solving various continuous problems, its binary version, BGSA, in solving binary encoded problems and its discrete version, DGSA, in solving combinatorial problems (Rashedi et al., 2010, 2011; Chatterjee and Mahanti, 2010; Yin et al., 2011; Sarafrazi et al., 2011; Bahrololoum et al., 2012; Li and Zhou, 2011; Taghipour et al., 2010; Dowlatshahi et al., 2014; Yazdani et al., 2014; Doraghinejad et al., 2014; Sarafrazi and Nezamabadi-pour, 2012, 2013; Rashedi et al., 2013). The advantages of GSA are the capability to escape from local optima and being easy to implement.

The outstanding results obtained by different variants of GSA encourage us to provide a Quantum-Inspired GSA (QIGSA) for binary encoded problems. In GSA all objects (agents) interact to each other to update the position (provide new solutions). This makes them to explore the search space comprehensively in addition to having good exploitation. Therefore, in the current work, our aim is to provide a quantum inspired version of GSA to effectively handle the binary encoded combinatorial problems; named as Binary Quantum-Inspired Gravitational Search Algorithm (BQIGSA). The main contributions of the paper are summarized as:

- A binary search algorithm, BQIGSA, by combination of GSA and QC is proposed. The proposed hybrid algorithm employs the stochastic characteristics of the individual described by a quantum system (e.g. Q-bit representation, superposition) and applies them to make an algorithm with more exploration capability.
- A modified rotation Q-gates strategy is suggested which causes the convergence to be occurred faster than usual.
- In contrast to other QIEAs, in the proposed method the basic concepts of the metaheuristic algorithm, GSA, such as acceleration, velocity and mass are used without no change.
- A comprehensive comparison with nine other binary algorithms is done and the results are presented.

The rest of the paper is organized as follows. Section 2 gives a review on the related works along with the basic concepts of QC. To make a proper background, a brief introduction to GSA and its binary version (BGSA) are presented in Section 3. The proposed BQIGSA is stated in Section 4. Experimental results and comparison with nine other algorithms are given on several cases of 0–1 knapsack problem, Max-Ones and Royal-Road functions in Section 5. Finally, the paper is concluded in Section 6.

2. Background

The metaheuristic search algorithm inspired by quantum mechanics, QIEAs, use quantum-inspired bits (Q-bits), quantum-inspired gates (Q-gates) and observation process to form their structure. This section first introduces important principles of quantum computing and then reviews the related works in the field of quantum-inspired evolutionary algorithms, especially, for solving binary-valued problems.

2.1. Quantum computing

The smallest unit of information that can be stored in a two-state quantum computer (Benioff, 1980) is called quantum bit (qubit/Q-bit) (Han and Kim, 2002). Quantum computers work by manipulating Q-bits and applying a sequence of quantum operators to them. A Q-bit is able to present states “1”, “0”, or any superposition of these two. The state of a Q-bit, $|\psi\rangle$, is presented by

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

where α and β are complex numbers that indicate probability amplitudes of the states “0” and “1”, respectively. The values $|\alpha|^2$ and $|\beta|^2$ are the probabilities that the Q-bit is found in the states “0” or “1”, respectively. Normalization requires that

$$|\alpha|^2 + |\beta|^2 = 1 \quad (2)$$

Therefore, a Q-bit is defined as a pair of number (α, β) as

$$q = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (3)$$

With regard to this definition, in a typical quantum system, an agent (individual/particle/object) is presented as a string with n Q-bits as follows (Han and Kim, 2002):

$$q = [q^1, q^2, \ldots, q^n] = \left[\begin{array}{c|c|c|c} \alpha^1 & \alpha^2 & \alpha^3 & \alpha^n \\ \beta^1 & \beta^2 & \beta^3 \cdots & \beta^n \end{array} \right] \quad (4)$$

where $|\alpha^d|^2 + |\beta^d|^2 = 1$ for $d = 1, 2, \ldots, n$. In general, a quantum system, $|\psi_n\rangle$, with n Q-bits can be in an arbitrary superposition of states simultaneously up to 2^n different states, as (Zhang, 2011)

$$|\psi_n\rangle = \sum_{i=1}^{2^n} C_i |S_i\rangle \quad (5)$$

where C_i is the probability amplitude of the state i, and S_i is described by the binary string $(x^1, x^2, \ldots, x^n)$,where $x^d \epsilon \{0, 1\}$, $d = 1, 2, \ldots, n$. In contrast, a normal computer can only be in one of these 2^n states at any one time. It is noticed that a Q-bit representation employs a probabilistic superposition of 0 and 1 and this representation extends naturally to multi-Q-bits systems (Zhang, 2011). However, in the act of observing a quantum state, it collapses to a single state. The pseudocode of observation process for dth Q-bit is given by Fig. 1. In this figure, *rand* [0,1] is a random number produced by a uniformly distributed probability function in the range [0, 1].

As an example, consider a three Q-bit system with three pairs of amplitude as follows:

$$\left[\begin{array}{c|c|c} 1/2 & 1/\sqrt{2} & -1/\sqrt{2} \\ -\sqrt{3}/2 & 1/\sqrt{2} & -1/\sqrt{2} \end{array} \right] \quad (6)$$

This system supports $2^3 = 8$ states including $|000\rangle$, $|001\rangle$, $|010\rangle$, $|100\rangle$, $|101\rangle$, $|011\rangle$, $|110\rangle$ and $|111\rangle$ with the probability amplitudes, C_i, $-1/4$, $-1/4$, $-1/4$, $\sqrt{3}/4$, $\sqrt{3}/4$, $-1/4$, $\sqrt{3}/4$ and $\sqrt{3}/4$, respectively. In other words, these states are presented by probabilities 1/16, 1/16, 1/16, 3/16, 3/16, 1/16, 3/16 and 3/16, respectively. Therefore, the advantage of Q-bit representation over classical representation is obvious from this example which can present each of eight possible states in the observation process with a specific probability.

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If      rand [0, 1] < (α^d)^2
Then    x^d = 0
Else    x^d = 1
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Fig. 1. The pseudocode of observation process in quantum-inspired algorithms.