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# Anti-windup based dynamic output feedback controller design with performance consideration for constrained Takagi–Sugeno systems

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ABSTRACT

This paper proposes an LMI-based method to guarantee the closed-loop regional stability and performance for Takagi–Sugeno systems that are subject to input saturation, state constraints, and also amplitude-bounded disturbance. Based on the Lyapunov stability tool, the proposed method provides conditions to simultaneously design the dynamic output feedback controller and its anti-windup compensator. By solving a convex optimization problem, these conditions are derived such that a tradeoff between the upper bound on the nominal $\mathscr{L}_2$ gain for exogenous disturbance and the minimal size of the domain of attraction can be found. This method is simple and systematic, allowing dealing with a very large class of constrained nonlinear systems. The effectiveness of the proposed method is illustrated with numerical examples.



## 1. Introduction

Over the past two decades, Takagi–Sugeno (T–S) fuzzy models (Takagi and Sugeno, 1985) have been intensively studied in the control community (Tanaka and Wang, 2001). It is motivated by the fact that these models have general approximation capability for complex dynamical system (Cao and Rees, 1997). Moreover, under weak conditions, a nonlinear model can be, globally or more often regionally, rewritten on a fuzzy T–S form. Stability analysis or controller synthesis is then facilitated due to their polytopic structure (Tanaka and Wang, 2001). As a consequence, this approach has now become a very attractive research topic in control theory (Feng, 2006). Stability analysis of a given T–S system is investigated in most cases via the direct Lyapunov method through the use of a quadratic Lyapunov function; the derived stability conditions are expressed as linear matrix inequalities (LMIs) (Boyd et al., 1994) for which efficient solvers are available. For the controller design, the choice of a parallel distributed compensation (PDC) control law is usually done for T–S models (Tanaka and Wang, 2001). An abundant literature is available on this nonlinear state feedback control law; see e.g. Feng (2006) for a quick overview.

For technological or economic reasons, the state variables are not all measured in most real-world applications. In order to deal with this practical problem, output feedback control must be used. In general, observer-based control scheme is proposed for unconstrained T–S systems (Liu and Zhang, 2003; Guerra et al., 2006). However, the design problem becomes much more complicated when state and/or input constraints have to be explicitly considered. As highlighted in Ding (2009), this control issue is not well addressed in the literature.

Due to physical/technical limitations and/or safety constraints, actuator saturation is unavoidable in almost all real applications. This phenomenon can severely degrade the closed-loop system performance and, in some cases, may lead to system instability. Motivated by this practical control aspect, a great deal of effort has been focused on saturated systems (Tarbouriech et al., 2011). In the literature, several methods now exist to handle saturation effects, but the most popular and effective one remains the anti-windup (AW) approach (see for instance Kothare et al. (1994)). LMI-based synthesis of anti-windup compensators has been proposed recently to synthesize either static (Gomes da Silva and Tarbouriech, 2005; Mulder et al., 2009) or dynamic anti-windup compensators (Cao et al., 2002; Grimm et al., 2003; Hu et al., 2008); an overview of these results can be found in the survey (Tarbouriech and Turner, 2009). Most of these works deal with a two-step method in which the controller and the AW strategy are designed separately. This method often proves to be satisfactory but it has however some drawbacks. First, only sub-optimal

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solution can be achieved since the controller and its AW compensator are designed separately (Sawada et al., 2009). Second, the effect of the nominal controller on the closed-loop performance under saturation is completely ignored (Mulder et al., 2009). To overcome these drawbacks, an alternative solution called one-step method, which designs simultaneously the controller and the AW compensator, has been proposed. Among the few works existing in the literature, we can notably cite (Mulder et al., 2009) in which an LMI solution ensuring global stability and performance is proposed for systems that are stable in open loop; Sawada et al. (2009) propose a regional approach based on a change of variable; whereas Dai et al. (2009) rely on the parameter elimination approach of Skelton et al. (1998). Both latter approaches are based on some preliminary results proposed in Hu et al. (2006).

To date, a large number of works on AW-based design are available for linear systems (Tarbouriech et al., 2011), but very few works deal with nonlinear cases, especially when exogenous disturbance signals are actively present (Gomes da Silva Jr. et al., 2013). In the T–S control framework, there are a couple of works devoted to the analysis or control design of saturated systems (for instance Cao and Lin (2003), Tseng and Chen (2006), Du and Zhang (2009), Bezzaoucha et al. (2013), Ariño et al. (2010)). However, very few papers are dedicated to AW synthesis for T–S systems: in Ting and Chang (2011), the authors addressed a two-step approach to deal with a continuous time-delay T-S systems; in Zhang et al. (2009), an interesting one-step approach based on piecewise fuzzy AW dynamic output feedback controller (DOFC) for discrete-time T–S systems has been proposed; note that these results seem to be valid only for systems that are stable in open-loop since no admissible sets of initial conditions are defined; and, at last, Song et al. (2011) which extends the approach proposed in Gomes da Silva and Tarbouriech (2005) to the case of T–S systems. It is noteworthy that an important point is neglected in all these results: besides control input saturation, the T–S model is only valid on a given subset of the state space. This is of course true for any model of real-world systems, but is fundamental in the writing of a T–S model using the nonlinear sector decomposition approach (Tanaka and Wang, 2001). This validity domain can be represented by some constraints on the state variables (see Example 2 for illustration of this fact). It is particularly important to consider explicitly these state constraints in the control design to ensure a good behavior of the closed-loop system in response to disturbances. This fact has been very recently emphasized in Nguyen (2013), Nguyen et al. (2014), Klug et al. (2015). The two first cited references concern stabilization of nonlinear switching systems under saturation. The latter one deals with the design of a dynamic output feedback control for nonlinear systems represented in Takagi–Sugeno form. Apart from the fact that we consider the effect of bounded disturbances on the design, a fundamental difference with the work in Klug et al. (2015) is that they considered discrete-time systems, whereas this paper concerns continuous-time systems. This allows them to use elegantly non-quadratic Lyapunov functions for the control law design. Multiple or non-quadratic Lyapunov functions are also developed in the continuous-time case, allowing one to obtain less conservative conditions for stability analysis (Tanaka et al., 2003; Mozelli et al., 2009). Concerning synthesis of control laws, the situation becomes much more complex and remains open due to the presence of the time-derivative of the membership functions in the stability conditions. Results thus far developed lead to very complex design conditions or to controllers requiring the inversion in real time of a parameter-dependent matrix (see for instances Guelton et al. (2009), Guerra et al. (2012), Bouarar et al. (2013)). The goal of this paper is to obtain a controller that may be easily implemented in practice. For this reason, the obtained results rely on the quadratic stabilization approach. However, as will be shown in Section 5 through numerical example, in the context of input-saturated T–S systems, the proposed results may be competitive in some sense with respect to those presented in Guerra et al. (2012) which actually provides the less-conservative results among all non-quadratic approaches found in the open literature.

In this paper, we address a novel one-step method to design simultaneously a DOFC and an AW compensator for a disturbed T–S system subject to control input and system state constraints. The proposed approach has some special features deserving particular attention: quadratic boundedness of the trajectories (see Ding (2009) and references therein) is ensured for any admissible initial condition and disturbance signal, as well as a maximal gain for the unsaturated system. Note that this performance may be ensured regionally for the saturated case as in Dai et al. (2009); however, this decoupling allows reducing the conservatism of the results. Moreover, it will be shown that the control design can be formulated as a multi-objective LMI optimization problem. In such a way, the obtained controller can solve the tradeoff between some predefined closed-loop requirements.

The paper is organized as follows. Section 2 describes the design problem and recalls some preliminaries results. The main result is stated in Section 3. In Section 4, a constructive control design is presented as a multi-objective LMI optimization problem. The results of the paper are effectively illustrated through an example in Section 5. Finally, Section 6 gives some concluding remarks.

The notations and terminology used in this paper are standard. For an integer number $r$, $\Omega_r$ denotes the set $\{1,2,\dots,r\}$. $\mathbb{R}^+=[0,\infty)$ is the set of non-negative real numbers. $\mathscr{L}_{2e}(\mathbb{R}^n)$ denotes the extended $\mathscr{L}_2$-space composed of measurable functions $f:[t_0,\infty)\mapsto\mathbb{R}^n$ such that $\int_{t_0}^{T}\|f(t)\|^2dt<\infty,\ \ \forall T\ge t_0$; and $\|f\|_{2,T}=\left(\int_{t_0}^{T}\|f(t)\|^2dt\right)^{1/2}$. $x_{(i)}$ is the $i$th element of a vector $x$. $x\succ y$, with $x,y\in\mathbb{R}^n$ means that $x_{(i)}-y_{(i)}>0$ for all $i\in\Omega_n$. $X_{(i)}$ denotes the $i$th row of a matrix $X$, and $sym(X)=X+X^T$ (for square matrices). $X>0$ means that $X$ is a symmetric, positive-definite matrix. $I$ denotes the identity matrix of appropriate dimension, and $(*)$ stands for matrix blocks that can be deduced by symmetry in a partitioned matrix. For $P\in\mathbb{R}^{n\times n}$ such that $P>0$, $\mathscr{E}(P)$ denotes the ellipsoid $\{x\in\mathbb{R}^n : x^TPx\le 1\}$. For any value of their arguments, the nonlinear functions $\eta_1,\dots,\eta_r$ are said to verify the convex sum property if $\eta_i\ge 0,\ \ \forall i\in\Omega_r$ and $\sum_{i=1}^{r}\eta_i=1$. The following notations are occasionally used:

$$Y_\theta=\sum_{i=1}^{r}\eta_i(\theta)Y_i;\quad Y_\theta^{-1}=\left(\sum_{i=1}^{r}\eta_i(\theta)Y_i\right)^{-1};\quad Z_{\theta\theta}=\sum_{i=1}^{r}\sum_{j=1}^{r}\eta_i(\theta)\eta_j(\theta)Z_{ij} \tag{1}$$

where $Y_i$, $Z_{ij}$ are matrices of appropriate dimensions and $\eta_i(\theta)$, $i\in\Omega_r$ are functions sharing the convex sum property.

## 2. Problem definition and preliminaries results

### 2.1. Control problem definition

#### 2.1.1. Closed-loop system description

Consider the following fuzzy T–S model described by (Tanaka and Wang, 2001) valid on a polyhedral domain:

$$\begin{cases} \dot{x}=\sum\limits_{i=1}^{r}\eta_i(\theta)\left(A_ix+B_i^uu+B_i^ww\right)\\ z=\sum\limits_{i=1}^{r}\eta_i(\theta)\left(C_i^zx+D_i^{zu}u+D_i^{zw}w\right)\\ y=\sum\limits_{i=1}^{r}\eta_i(\theta)\left(C_i^yx+D_i^{yw}w\right)\end{cases} \tag{2}$$

where $x\in\mathcal{P}_x\subset\mathbb{R}^{n_x}$, $u\in\mathbb{R}^{n_u}$, $w\in\mathbb{R}^{n_w}$, $y\in\mathbb{R}^{n_y}$ and $\theta\in\mathbb{R}^k$ are respectively, the state, the control input, the disturbance, the measured output and the scheduling variable vectors of the system. The regulated output vector $z\in\mathbb{R}^{n_z}$ is used for performance purposes. For $i\in\Omega_r$, the real constant matrices of appropriate dimensions $A_i$,