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Gain estimation of nonlinear dynamic systems modeled by an FBFN and the maximum output scaling factor of a self-tuning PI fuzzy controller

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ABSTRACT

This paper proposes new techniques to calculate the dynamic gains of nonlinear systems represented by fuzzy basis function network (FBFN) models. The dynamic gain of an FBFN can be approximated by finding the maximum of norm values of the locally linearized systems or by solving a non-smooth optimal control problem. From the proposed gain calculation techniques, a novel adaptive multilevel fuzzy controller (AMLFC) with a maximum output scaling factor is presented. To guarantee the system stability, a stability condition is derived, which only requires that the output scaling factor of the AMLFC be bounded. Therefore, this paper provides a systematic and simple design practice for controlling nonlinear systems by using an AMLFC. The AMLFC is simulated in a tower crane control system. Simulation results show that AMLFC is not only robust but also provides improved transient performances compared with the robust adaptive fuzzy controller.

1. Introduction

Fuzzy controllers are constructed based on heuristic rules and "expert knowledge" derived from physical systems. Early fuzzy control papers did not provide mathematical stability analysis or proofs of the control systems (Sala, 2013). However, the stability of a fuzzy control system is very important in the controller design process to guarantee desired performance and safety in the plant operations.

The applications of the small gain theorem (Jiang et al., 2010; Yang, 2005) and the passivity theory (Xu and Shin, 2005) in fuzzy control systems show great advantages compared to other stability methods. These stability theories do not require an exact mathematical representation of the plant and, therefore, they can be applied to nonlinear systems with unknown mathematical models. With the small gain theorem, Chen and Ying (1993) demonstrated how the parameters of a proportional-integral (PI) fuzzy controller could be chosen to ensure the input-output stability of a nonlinear system. However, the stability criteria developed are only limited to a certain type of fuzzy controllers with two input and three output membership functions. Since Chen and Ying (1993) divided the stability problem according to the locations of the error and the time rate of change of the error with respect to zero, the complexity of the problem would exponentially increase if the number of input and output membership functions increases. In the current work, the stability analysis is conducted based on the location of the error and the time rate of change of the error with respect to the activated membership functions. The results, therefore, can be applied to fuzzy controllers with any number of input and output membership-functions.

Since mathematical models for nonlinear systems cannot always be easily obtained, fuzzy basis function network (FBFN) models were adopted in many applications (Chiang et al., 2012; Lee and Shin, 2001; Leng et al., 2005) to represent the relationship between the inputs and outputs of the systems. With a set of input and output data, Wang and Mendel (1992) showed that any nonlinear system can be approximated by an FBFN model. However, controllers implemented with FBFN models are still limited, owing to a lack of stability analysis. Due to the nonlinearity characteristics of the FBFN, the small gain theorem is the most appropriate approach to finding the stability region in this case. Therefore, obtaining the dynamic gain from an FBFN model is the first step towards achieving the stability condition for nonlinear fuzzy control systems.

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In many applications where heuristic information for designing a fuzzy controller is not sufficient, the parameters of a fuzzy controller can be computed offline by using input and output data (Chen et al., 2009; Lin and Xu, 2006; Mingzhi et al., 2009). Ying (1994) introduced a method for obtaining the parameters of a PI fuzzy controller by tuning a linear PI controller. However, the global stability of the control system could not be guaranteed, since Ying's method only showed local stability around the equilibrium points, nor could it determine the size of the region of local stability. When there are disturbances and time-varying parameters, online adaptation of control parameters based on data gathered in real-time would be more effective. Li and Tong (2003) proposed a hybrid control system, which consists of a state observer, an adaptive fuzzy mechanism, an H^{∞} control and a sliding mode control. Boubakir et al. (2011) used a different approach to tune the parameters of a proportional-integral-derivative (PID) controller for multi-input multi-output (MIMO) dynamic systems by minimizing the error between an ideal controller and the PID controller. However, the controllers developed by both Li and Tong (2003) and Boubakir et al. (2011) can only be applied to a certain class of nonlinear dynamic systems where the input is represented by a linear term in the system's mathematical model. Pellegrinetti and Bentsman (1996) offer an example of nonlinear systems that cannot be represented in this form. Furthermore, stability conditions for the controllers presented in these papers must be calculated based on the upper bounds of the model functions. These values are difficult to obtain in many cases where the system models are unknown. In the current work, since an FBFN is used as a representation of nonlinear systems, the stability condition depends only on the dynamic gain that can be computed directly from the FBFN's parameters.

Different studies have been conducted to improve the performance of fuzzy controllers. Haj-Ali and Ying (2004) and Arya (2007) have analyzed the structures of PI fuzzy controllers and found the effects of nonlinear and asymmetrical input sets on the performance of the controllers. Chen and Ying (1993) and Haj-Ali and Ying (2004) demonstrated that fuzzy PI and PID controllers could be treated as nonlinear PI and PID controllers. Mudi and Pal (1999) presented a method to tune the output-scaling factors of fuzzy controllers by using the error and the time rate of change of the error signals. However, this method is based only on an intuitive analysis of the desired performances to keep the system stable; no mathematical stability analysis was provided in their work. In Woo et al. (2000), a PID fuzzy controller was proposed with self-tuning algorithms for both input and output scaling factors, but lacked a systematic stability analysis. The multilevel fuzzy controller (MLFC) system was proposed by Xu and Shin (2005), wherein the controller has an adaptive mechanism designed to tune the output membership functions based on the system outputs. Although the MLFC has been successfully utilized in different applications (Davis et al., 2011; Ngo and Shin, 2012), the controller still has some limits when dealing with time-variant systems such as sectorial restrictions on membership functions.

The current work proposes a novel method to estimate the dynamic gain of a nonlinear system and discusses the design process for a new MLFC with an adaptive mechanism for the output scaling factor. The design can improve the transient performance of control systems while eliminating the need for initial parameter tuning. The stability analysis is conducted based on the small gain theorem and uses the dynamic gain of the nonlinear system to provide the maximum bound of the MLFC's output scaling factor for system stability.

2. Dynamic gain estimation of nonlinear dynamic systems modeled by FBFNs

The stability analysis of a nonlinear fuzzy control system based on the small gain theorem requires an estimation of the dynamic gain of the plant. Two methods are provided in this section to calculate the gain of an FBFN system. In the first method, the dynamic gain can be approximated by finding the maximum of the norm values of the locally linearized systems. This method provides an effective technique for FBFN models with a large number of fuzzy rules, since the estimation can be done based on experimental data. The second method provides an analytical computation technique of the dynamic gain based on a non-smooth optimal control problem. To simplify mathematical analysis, only nonlinear systems with single input and single output (SISO) are considered in this paper. However, the technique can be easily expanded to MIMO systems by applying the same procedure for each individual input and output pair.

2.1. Local linear model of a nonlinear systems represented by FBFNs

This subsection provides a method for obtaining the local linear model of a nonlinear system from its FBFN model. For a SISO nonlinear system, an FBFN model can be constructed from the input and output data through a set of l fuzzy rules, where the ith rule R^i is described as following:

$$\begin{aligned}&\mathrm{R}^i: \quad \text{If } u(k-1)=A_1^i \text{ AND } u(k-2)=A_2^i \text{ AND} \ldots \text{AND } u(k-m)=A_m^i \text{ AND}\\ &y(k-1)=B_1^i \text{ AND } y(k-2)=B_2^i \cdots \text{AND } y(k-n)=B_n^i\\ &\text{then } y(k)=b^i\end{aligned} \tag{1}$$

where $u(k)$ is the input and $y(k)$ denotes the output of the nonlinear system at time instance k, m and n represent the system orders of the input and the output,

$A_1 \ldots A_m$ and $B_1 \ldots B_n$ are fuzzy membership sets, and

b represents a singleton function of the output.

Assume that the output of the FBFN model at initial condition is zero, by using singleton fuzzification, product inference and centroid defuzzification methods, the FBFN model can be represented by the following state space equations:

$$\begin{aligned}&\mathbf{x}(k)=\mathbf{f}(\mathbf{x}(k-1),\mathbf{u}(k-1))\\ &y(k)=\mathbf{c}^T\mathbf{x}(k)\end{aligned} \tag{2}$$

where

$$\mathbf{x}(k)=\left[y(k),\ldots y(k-n+1)\right]^T,\ \mathbf{u}(k)=\left[u(k),\ldots,u(k-m+1)\right]^T,\ \mathbf{x}(0)=[0,\ 0,\ \ldots,0]^T, \tag{3}$$