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An opposition-based algorithm for function optimization

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ABSTRACT

The concept of opposition-based learning (*OBL*) was first introduced as a scheme for machine intelligence. In a very short period of time, some other variants of opposite numbers were proposed and opposition was applied to various research areas. In metaheuristic optimization algorithms, the main idea behind applying opposite numbers is the simultaneous consideration of a candidate solution and its corresponding opposite candidate in order to achieve a better approximation for the current solution. This paper proposes an opposition-based metaheuristic optimization algorithm (*OBA*) and a new and efficient opposition named comprehensive opposition (*CO*) as its main operator. In this paper it is mathematically proven that *CO* not only increases the chance of achieving better approximations for the solution but also guarantees the global convergence of *OBA*. The efficiency of the proposed method has been compared with some well-known heuristic search methods. The obtained results confirm the high performance of the proposed method in solving various function optimizations.

1. Introduction

Metaheuristics have been established as one of the most practical approaches to optimization problems. They have been primarily designed to address problems that cannot be tackled through traditional optimization algorithms. Although still there is no guarantee, metaheuristic methods usually turn out to achieve better results and better performances in contrast to their classic counterparts. Popular metaheuristic optimizers include genetic algorithm (Holland, 1992), particle swarm optimization (Kennedy and Eberhart, 1995), differential evolution (Storn and Price, 1997), evolution strategies (Rechenberg, 1971), ant colony optimization (Dorigo and Di Caro, 1999), gravitational search algorithm (Rashedi et al., 2009), etc. These algorithms solve different optimization problems. However, there is no specific algorithm to achieve the best solution for all optimization problems. Some algorithms give a better solution for some particular problems than others. Hence, searching for new metaheuristic optimization algorithms is always needed (Wolpert and Macready, 1997).

Recently, results for metaheuristic optimization algorithms have been reported which indicate that the simultaneous consideration of randomness and opposition is more advantageous than pure randomness (Rahnamayan and Tizhoosh, 2008; Rahnamayan et al., 2012; Ventresca et al, 2010). This new scheme, called opposition-based learning, has an apparent effect on accelerating metaheuristic optimization algorithms. The concept of opposition-based learning (*OBL*) was firstly introduced by Tizhoosh (Tizhoosh, 2005a). In a very short period of time, some other variants of opposition-based learning such as quasi-opposition (*QO*) (Rahnamayan et al., 2007), quasi-reflection (*QR*) (Ergezer et al., 2009) and current optimum opposition (*COOBL*) (Xu et al., 2011) were applied to various research areas. The achieved empirical results confirm that the concept of opposition is general enough and can be utilized in a wide range of learning and optimization fields.

OBL was firstly proposed as a machine intelligence scheme for reinforcement learning (Tizhoosh, 2005a, 2005b, 2006). Afterward, it has been employed to enhance soft computing methods such as fuzzy systems (Tizhoosh, 2009; Tizhoosh and Sahba, 2009) and artificial neural networks (Rashid and Baig, 2010; Shokri et al., 2007; Ventresca and Tizhoosh, 2006; Ventresca and Tizhoosh, 2007a, 2008). *OBL* has been proven to be an effective method for solving optimization problems. It has been shown that in terms of convergence speed, utilizing *OBL* is more beneficial than using the pure randomness to generate initial estimates for a population based metaheuristic optimization algorithm in the absence of a prior knowledge about the solution of a box-constrained continuous domain optimization problem (Rahnamayan et al., 2008a, 2012; Ventresca et al., 2010). *OBL* has been employed to improve the success rate of various evolutionary algorithms such as differential evolution (Rahnamayan, 2008; Rahnamayan et al., 2006a, 2006b, 2008b, 2008c, 2008d; Rahnamayan and Wang, 2008a, 2008b), particle swarm optimization (Chi and Cai, 2010; Han and He, 2007; Jabeen et al., 2009; Omran, 2009; Shahzad et al., 2009; Tang and Zhao, 2009; Wang et al., 2007), ant colony optimization (Malisia, 2007, 2008; Malisia and Tizhoosh, 2007), simulated annealing (Ventresca and Tizhoosh, 2007b), harmony search

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(Mahamed et al., 2011), biogeography-based optimization (Bhattacharya and Chattopadhyay, 2010) and gravitational search algorithm (Shaw et al., 2012) in a wide range of fields from image processing (Khalvati et al., 2007; Rahnamayan and Tizhoosh, 2008) to system identification (Subudhi and Jena, 2009; Subudhi and Jena, 2011). Enhancing searching or learning in different fields was tried using all of these algorithms and they were experimentally verified by benchmark functions. A majority of these algorithms and also other opposition-based works have been explained in Tizhoosh et al. (2008). In 2007, *QO* was proposed by Rahnamayan et al. (Rahnamayan et al., 2007). It was successfully used in differential evolution (Peng and Wang, 2010; Rahnamayan et al., 2007), and particle swarm optimization (Zhang et al., 2009). Since a candidate solution is reflected to its opposite to accelerate the exploration, Mehmet Ergezer et al. applied the same logic and reflected the quasi-opposite point to obtain the *QR* point (Ergezer et al., 2009). It was applied on biogeography-based optimization (Ergezer et al., 2009). Recently, *COOBL* has been proposed and combined with differential evolution for function optimization (Xu et al., 2011). The opposite points created using the current optimum are in the neighborhood of the best solution that has been found during the process, especially in later stages.

In this paper, first we introduce two new oppositions named extended opposition (*EO*) and reflected extended opposition (*REO*) and apply them as well as *QO* and *QR* to propose comprehensive opposition (*CO*). To increase the probability of achieving better approximations for the optimal solution while controlling the diversity of candidate solutions, *CO* shifts the amount of each variable $x\epsilon[a,b]$ to one of its opposite points $\breve{x}^{reo}$, $\breve{x}^{qr}$, $\breve{x}^{qo}$ or $\breve{x}^{eo}$ whose probabilities of being selected are the optimal solution of a linear parametric programming with parameter t/T where t is the number of iteration and T is the total number of iterations. Then, we introduce an optimization algorithm named opposition based algorithm (*OBA*) that employs *CO* as its main operator. We use probability rules to analyze the effect of *CO* on the optimization performance. We also model *OBA* as a Markov chain and show that *OBA* converges asymptotically with probability one to a global optimum. The most significant advantages of the proposed algorithm are its strong mathematical base and its guaranteed convergence.

This paper is organized as follows. Section 2 provides a brief review of opposition and its features and then introduces *EO* and *REO*. Finally, it covers the definitions, theorems, and proofs corresponding to *CO*. *OBA* is described in Section 3. In Section 4, *OBA* is modeled as a Markov chain and its global convergence is established. A comparative study is presented in Section 5. An experimental study is given in Section 6 and the performance of *OBA* algorithm will be evaluated on nonlinear benchmark functions and the results are compared with those of *PSO* and *GSA*. Finally, a conclusion is given in Section 7.

2. Opposition in box-constrained optimization problems

Opposition-based learning is a concept firstly proposed in computational intelligence (Tizhoosh, 2005a), and has been proven to be an effective concept to enhance various metaheuristic optimization algorithms. When evaluating a solution X to a given problem, simultaneously computing its opposite solution will provide another chance for finding a candidate solution closer to the global optimum. Opposition-based learning has different variants employed for solving box-constrained optimization problems. These variants include opposition (Tizhoosh, 2005a), quasi-opposition (Rahnamayan et al., 2007), quasi reflection (Ergezer et al., 2009) and current optimum opposition (Xu et al., 2011). In this section, first the concepts of these variants are reviewed and then extended opposition and reflected extended opposition are introduced. Finally, quasi-opposition, quasi reflection, extended opposition and reflected extended opposition are applied to propose comprehensive opposition.

2.1. Opposition

In opposition, the amount of each variable is reflected through the center of its domain to create its opposite number as defined below (Tizhoosh, 2005a).

Definition 1. Let $X(x_1, x_2, \ldots, x_d)$ be a point in d-dimensional space, where $x_1, x_2, \ldots, x_d$ are real numbers and $x_i \in [a_i, b_i]$, $i = 1, .., d$. The opposite point of X is denoted by $\breve{X}(\breve{x}_1, \breve{x}_2, \ldots, \breve{x}_d)$ where $\breve{x}_i = a_i + b_i - x_i$, $i = 1, \ldots, d$.

2.2. Quasi-opposition

Quasi-opposition, defined below, shifts the amount of each variable to a random point between the center of its domain and its opposite number (Rahnamayan et al., 2007).

Definition 2. Let $X(x_1, x_2, \ldots, x_d)$ be a point in d-dimensional space, where $x_1, x_2, \ldots, x_d$ are real numbers and $x_i \in [a_i, b_i]$, $i = 1, .., d$. The quasi-opposite point of X is denoted by $\breve{X}^{qo}(\breve{x}_1^{qo}, \breve{x}_2^{qo}, \ldots, \breve{x}_d^{qo})$ where $\breve{x}_i^{qo} = rand(c_i, \breve{x}_i)$, $c_i = (a_i + b_i)/2$, $i = 1, \ldots, d$.

2.3. Quasi-reflection

Quasi-reflection, defined below, reflects the amount of each variable x to a random point between the center of its domain and x (Ergezer et al., 2009).

Definition 3. Let $X(x_1, x_2, \ldots, x_d)$ be a point in d-dimensional space, where $x_1, x_2, \ldots, x_d$ are real numbers and $x_i \in [a_i, b_i]$, $i = 1, .., d$. The quasi-reflected point of X is denoted by $\breve{X}^{qr}(\breve{x}_1^{qr}, \breve{x}_2^{qr}, \ldots, \breve{x}_d^{qr})$ where $\breve{x}_i^{qr} = rand(x_i, c_i)$ and $c_i = (a_i + b_i)/2$ for $i = 1, \ldots, d$.

2.4. Current optimum opposition

The opposite point using the current optimum is in the neighborhood of the global optimum during the process of evolution, especially in later stages (Xu et al., 2011).

Definition 4. Let $X(x_1, x_2, \ldots, x_d)$ be a point in d-dimensional search space, $X^{best}(x_1^{best}, x_2^{best}, \ldots, x_d^{best})$ be the best solution in the current population, and x_i, $x_i^{best} \in [a_i, b_i]$, $i = 1, .., d$. The current optimum opposite point of X is denoted by $\breve{X}^{coo}(\breve{x}_1^{coo}, \breve{x}_2^{coo}, \ldots, \breve{x}_d^{coo})$ where $\breve{x}_i^{coo} = 2x_i^{best} - x_i$, $i = 1, \ldots, d$. Here X^{best} is the center of opposition.

It is possible that the opposite candidate $\breve{x}_i^{coo}$, $1 \le i \le d$, jumps out of the box constraint $[a_i, b_i]$. In this case, the opposite candidate is assigned to a value as follows:

$$\breve{x}_i^{coo} = \begin{cases} a_i, & \breve{x}_i^{coo} < a_i \\ b_i, & \breve{x}_i^{coo} > b_i \end{cases}.$$

2.5. Extended opposition

We introduce extended opposition as follows.

Definition 5. Let $X(x_1, x_2, \ldots, x_d)$ be a point in d-dimensional space, where $x_1, x_2, \ldots, x_d$ are real numbers and $x_i \in [a_i, b_i]$, $i = 1, .., d$. The extended opposite point of X is denoted by $\breve{X}^{eo}(\breve{x}_1^{eo}, \breve{x}_2^{eo}, \ldots, \breve{x}_d^{eo})$